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8. Exercises for the Course „Logic-based Knowledge Representation“

Exercise 30:

For the following $\mathcal{FL}_0$-concepts, show that C is *not* subsumed by D by giving an interpretation $\mathcal{I}$ such that $C^{\mathcal{I}} \not\subseteq D^{\mathcal{I}}$:

- $C = \forall r.\forall s.(A \sqcap B) \sqcap \forall r.(A \sqcap \forall s.\forall t.A), \quad D = \forall r.(A \sqcap \forall s.(A \sqcap B \sqcap \forall t.A \sqcap \forall r.B))$
- $C = \forall r.(A \sqcap \forall s.B) \sqcap \forall r.(B \sqcap \forall s.\forall t.A), \quad D = \forall r.(A \sqcap B) \sqcap \forall s.\forall r.A \sqcap \forall r.\forall s.B \sqcap \forall r.(\forall s.B \sqcap \forall t.A)$

Exercise 31:

As discussed in the lecture, the co-NP-hardness of subsumption in $\mathcal{FL}_0$ w.r.t. acyclic TBoxes can be proved by reducing the containment problem of acyclic finite automata, which is well-known to be co-NP-complete. Prove that subsumption in $\mathcal{FL}_0$ w.r.t. acyclic TBoxes is in co-NP (thus co-NP-complete) by reversing this reduction: devise a polynomial reduction from the subsumption problem of $\mathcal{FL}_0$ w.r.t. acyclic TBoxes to containment of acyclic finite automata.

Hints:

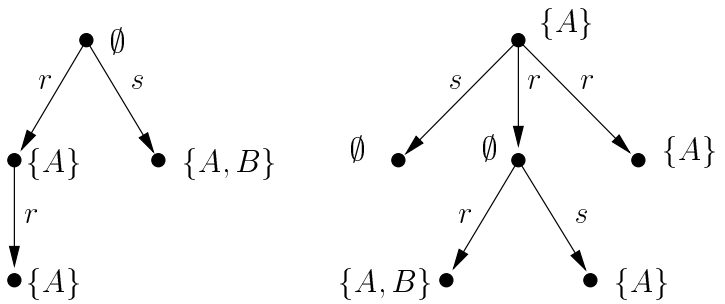
- Devise a normal form $\forall \mathcal{A}_1.A_1 \sqcap \cdots \sqcap \forall \mathcal{A}_n.A_n$ for $\mathcal{FL}_0$-concepts that takes into account TBoxes. In the normal form, the $\mathcal{A}_i$ are acyclic finite automata (instead of sets of words).
- To generate the automata $\mathcal{A}_i$, translate the *defined* concept names from the TBox into automata states. Add additional states if necessary.
- Reduce the subsumption of concepts in the new normal form to the containment problem of acyclic finite automata (for each primitive concept name in the input, one containment problem has to be solved).

Exercise 32:

Convert the following $\mathcal{EL}$-concepts into description trees:

- $\top$
- A
- $A \sqcap \exists R.\exists S.\top$
- $\exists R.\exists S.A \sqcap \exists T.B \sqcap \exists T.A$
- $B \sqcap \exists R.(A \sqcap B \sqcap \exists S.(A \sqcap B) \sqcap \exists T.A)$

Convert the following description trees into $\mathcal{EL}$-concepts:



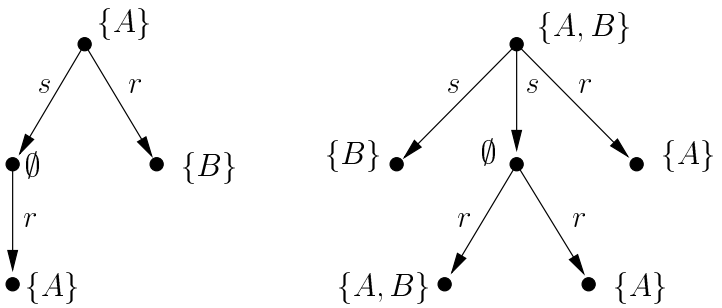
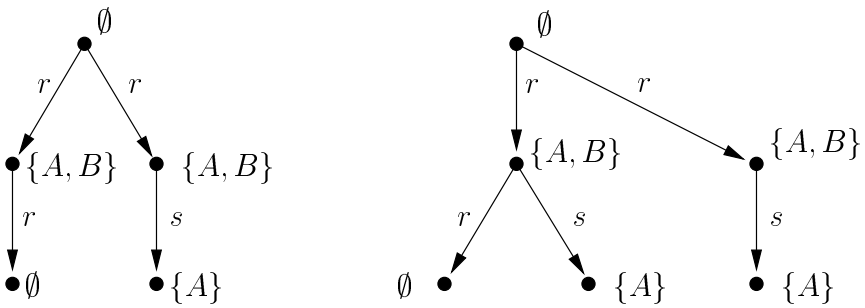
Exercise 33:

In the lecture, the translation of $\mathcal{EL}$-concepts into description trees has been presented on an intuitive level, only. Define this translation formally using induction on the structure of $\mathcal{EL}$-concept trees.

Exercise 34:

Compute all possible homomorphisms between the following description trees:

1.



2.