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9. Exercises for the Course „Logic-based Knowledge Representation“

Exercise 35:

Let $\mathcal{L}$ be a Description Logic and C, D, and E be $\mathcal{L}$-concept terms. Prove that the following properties hold:

(a) $\mathrm{lcs}(C, D) = \mathrm{lcs}(D, C)$

(b) $\mathrm{lcs}(C, \mathrm{lcs}(D, E)) = \mathrm{lcs}(\mathrm{lcs}(C, D), E)$

(c) The least common subsumer of C and D is unique up to equivalence.

Exercise 36:

Compute the least common subsumer of

(a) the following pairs of $\mathcal{FL}_0$-concept terms:

$$C = A_1 \sqcap A_2 \sqcap \forall r.(A_1 \sqcap \forall s.A_1 \sqcap \forall r.A_2) \sqcap \forall r.\forall s.\forall r.(A_1 \sqcap A_2 \sqcap \forall s.A_2)$$
$$D = A_2 \sqcap \forall r.\forall r.(A_2 \sqcap A_3) \sqcap \forall s.A_2 \sqcap \forall r.(A_2 \sqcap \forall s.(A_2 \sqcap \forall r.(A_2 \sqcap A_3)))$$

(b) the following pairs of $\mathcal{EL}$-concept terms:

$$C = A \sqcap \exists r.(A \sqcap \exists(A \sqcap B)) \sqcap \exists r.(B \sqcap \exists s.B) \sqcap \exists s.(A \sqcap \exists r.\top)$$
$$D = B \sqcap \exists r.(A \sqcap B \sqcap \exists s.(A \sqcap B \sqcap \exists s.(A \sqcap B))) \sqcap \exists s.(A \sqcap B) \sqcap \exists s.(B \sqcap \exists r.\top)$$

Exercise 37:

Consider the following $\mathcal{EL}$-concept terms:

$$C_1 = \exists r.(A \sqcap \exists r.(A \sqcap B \sqcap \exists r.(A \sqcap B))) \sqcap \exists r.(B \sqcap \exists r.(A \sqcap B \sqcap \exists r.(A \sqcap B)))$$
$$C_2 = \exists r.(A \sqcap B \sqcap \exists r.(A \sqcap \exists r.(A \sqcap B)) \sqcap \exists r.(B \sqcap \exists r.(A \sqcap B)))$$
$$C_3 = \exists r.(A \sqcap B \sqcap \exists r.(A \sqcap B \sqcap \exists r.A \sqcap \exists r.B))$$

Compute $D = \mathrm{lcs}(C_1, \mathrm{lcs}(C_2, C_3))$ and prove that there is no $\mathcal{EL}$-concept term E such that D and E are equivalent and E's length is smaller than $2^4 - 1$ (i.e., the number of nodes in a tree of depth 3).

Exercise 38:

Let $\mathcal{EL}_\neg$ be the extension of $\mathcal{EL}$ obtained by allowing the use of negation in front of concept *names*. For example, $A \sqcap \exists r.(B \sqcap \neg A)$ is an $\mathcal{EL}_\neg$-concept term, but $A \sqcap \neg(\exists r.(A \sqcap B))$ is not.

Devise an algorithm for computing the lcs of two $\mathcal{EL}_\neg$-concept terms.

Hint: First try to understand when an $\mathcal{EL}_\neg$-concept term is unsatisfiable.