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11. Exercises for the Course „Logic-based Knowledge Representation“

Exercise 42:

Choose a technique to decide whether propositional logic formulas are valid and use it to decide validity of the following formulas:

(a) $\neg(a \wedge b) \rightarrow (\neg(a' \wedge \neg b) \rightarrow \neg(a \wedge a'))$

(b) $(a \rightarrow (b \rightarrow c)) \rightarrow ((c \rightarrow d) \rightarrow (a \rightarrow (b \rightarrow d)))$

(c) $a \vee (c \rightarrow b) \vee (a \wedge \neg d) \vee (d \wedge \neg c \wedge \neg f) \vee ((c \vee b) \wedge (\neg(g \vee e))) \vee (g \wedge a \wedge b) \vee (e \wedge d \wedge \neg c)$

Hint: Choose an *appropriate* technique (since the above formulas contain many variables, truth tables may not be approppriate).

Exercise 43:

Prove that the following K_n-formulas are theorems:

(a) $[n]([m](p \vee q) \vee \neg[m](p \vee q))$

(b) $[m](p \wedge q) \rightarrow [m]q$ and $[m](p \wedge q) \rightarrow [m]p$

(c) $[m](p \wedge q) \rightarrow ([m]p \wedge [m]q)$

Exercise 44:

Let Δ and Γ be sets of K_n-formulas. Prove or refute the following claims:

(a) if Γ is consistent and $\Delta \subseteq \Gamma$, then Δ is consistent

(b) if Γ is inconsistent and $\Gamma \subseteq \Delta$, then Δ is inconsistent.

(c) Let $\Gamma = \bigcup_{i \geq 0} \Gamma_i$. Then Γ is consistent iff all Γ_i are consistent.

(d) Let $\Gamma = \bigcup_{i \geq 0} \Gamma_i$ and $\Gamma_0 \subseteq \Gamma_1 \subseteq \Gamma_2 \cdots$. Then Γ is consistent iff all Γ_i are consistent.

Exercise 45:

Prove Lemma 7.16 from the lecture:

Lemma 7.16 Let R_m be a reachability relation.

(a) If R_m is symmetric and transitiv, then R_m is euklidian.

(b) The following three conditions are equivalent:

(i) R_m is symmetric, transitiv, and seriell.

(ii) R_m is reflexiv and euklidian.

(iii) R_m is an equivalence relation, i.e., reflexiv, transitiv, and symmetric.