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13. Exercises for the Course „Logic-based Knowledge Representation“

Exercise 50:

Consider the modal logic $\mathsf{S5}_1$ (i.e., $\mathsf{S5}_n$ with a single modality). Prove the following:

(a) If a (uni-)modal formula φ is satisfiable in a Kripke structure whose accessibility relation is universal (see Exercise 48) then it is satisfiable in a Kripke structure $\mathcal{K} = (W, \pi, R)$ such that R is universal and the cardinality of W is at most polynomial in the length of φ.

(b) It is in NP to decide whether, for a given (uni)-modal formula φ, it holds that $\vdash_{\mathsf{S5}_1} \varphi$.

Exercise 51:

(a) Let $D := \left\{ \frac{\text{Italian}(x) : \text{lovesWine}(x)}{\text{lovesWine}(x)}, \ \frac{\text{French}(x) : \text{lovesWine}(x)}{\text{lovesWine}(x)} \right\}$

$W := \{\text{Italian(Tom)} \vee \text{French(Tom)}\}$.

Compute the extensions of D and W and decide whether Tom loves wine.

(b) Let $D := \left\{ \frac{\text{true} : \text{usable}(x) \wedge \neg\text{broken}(x)}{\text{usable}(x)} \right\}$

$W := \{\text{broken(leftArm)} \vee \text{broken(rightArm)}\}$.

Compute the extensions of D and W and decide whether both arms are usable.

Exercise 52:

Consider the default theory (D, W) from Example 8.5 of the lecture: for closed atomic formulae a, b, c of first order predicate logic, we define

$$D := \left\{ \frac{a : b}{c}, \ \frac{c : \neg b}{\neg b} \right\}$$

$$W := \{a\}.$$

Prove that (D, W) has no extension.

Exercise 53:

Let (D, W) be a default theory.

- Let $\mathcal{E}_{(D,W)}$ be the set of all extensions of (D, W).
- We call $E, E' \in \mathcal{E}_{(D,W)}$ *incomparable*, if neither $E \subseteq E'$ nor $E' \subseteq E$.

Prove of disprove the following claims:

(a) There exists a default theory (D, W) with $|\mathcal{E}_{(D,W)}| \geq 2$ and where all $E, E' \in \mathcal{E}_{(D,W)}$ with $E \neq E'$ are incomparable.

(b) There exists a default theory (D, W) with $|\mathcal{E}_{(D,W)}| \geq 2$ and where, for all $E, E' \in \mathcal{E}_{(D,W)}$, either $E \subseteq E'$ or $E' \subseteq E$.