Proof. of Lemma 3.14

Point 1 can be proved using the following derivation:

(1)	$\vdash_L \varphi \to \psi$	given
(2)	$\vdash_L \Box(\varphi \to \psi)$	Gen.
(3)	$\vdash_L \Box(\varphi \to \psi) \to (\Box\varphi \to \Box\psi)$	K + Subst.
(4)	$\Box\varphi \to \Box\psi$	(2) + (3) + MP

To prove 2, we first show

$$\vdash_L (\Box\varphi \land \Box\psi) \to \Box(\varphi \land \psi) \qquad (*)$$

using the following derivation:

(1)	$\vdash_L \varphi \to (\psi \to (\varphi \land \psi))$	P.T. $a \to (b \to (a \land b))$ + Subst.
(2)	$\vdash_L \Box\varphi \to \Box(\psi \to (\varphi \land \psi))$	(1) + Point 1
(3)	$\vdash_L \Box(\psi \to (\varphi \land \psi)) \to (\Box\psi \to \Box(\varphi \land \psi))$	K + Subst.
(4)	$\vdash_L \Box\varphi \to (\Box\psi \to \Box(\varphi \land \psi))$	P.T. $(a \to b) \to ((b \to c) \to (a \to c))$ + Subst. + (2) + (3) + 2 * MP
(5)	$\vdash_L (\Box\varphi \land \Box\psi) \to \Box(\varphi \land \psi)$	P.T. $(a \to (b \to c)) \to ((a \land b) \to c)$ + Subst. + (4) + MP

Using induction on n, we can now prove Point 2:

- $n = 2$. Immediate from $(*)$.
- $n > 2$.

(1)	$\vdash_L (\Box\psi_2 \land \cdots \land \Box\psi_n) \to \Box(\psi_2 \land \cdots \land \psi_n)$	IH
(2)	$\vdash_L (\Box\psi_1 \land \Box(\psi_2 \land \cdots \land \psi_n)) \to \Box(\psi_1 \land \cdots \land \psi_n)$	$(*)$
(3)	$\vdash_L (\Box\psi_1 \land \cdots \land \Box\psi_n) \to \Box(\psi_1 \land \cdots \land \psi_n)$	

Last line: P.T. $(b \to c) \to (((a \land c) \to d) \to ((a \land b) \to d))$

$$\begin{array}{lrcl} + \text{ Subst.} & a & := & \Box\psi_1 \\ & b & := & \Box\psi_2 \land \cdots \land \Box\psi_n \\ & c & := & \Box(\psi_2 \land \cdots \land \psi_n) \\ & d & := & \Box(\psi_1 \land \cdots \land \psi_n) \end{array}$$

+ (1) + (2) + 2 * MP

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