(Partial) Proof. of the Existence Lemma

$$
\begin{array}{lll}
(1) & \vdash_L \neg(\psi_1 \wedge \cdots \wedge \psi_n) & \text{for some } \psi_1, \ldots, \psi_n \in v^- \\
(2) & \vdash_L \psi_1 \wedge \cdots \wedge \psi_n \to \neg\varphi & \text{P.T. } \neg a \to (a \to \neg b) + \text{Subst} + \text{MP} \\
(3) & \vdash_L \Box(\psi_1 \wedge \cdots \wedge \psi_n \to \neg\varphi) & \text{Gen.} \\
(4) & \vdash_L \Box(\psi_1 \wedge \cdots \wedge \psi_n \to \neg\varphi) & \mathsf{K} + \text{Subst} \\
 & \qquad \to (\Box(\psi_1 \wedge \cdots \wedge \psi_n) \to \Box\neg\varphi) & \\
(5) & \vdash_L \Box(\psi_1 \wedge \cdots \wedge \psi_n) \to \Box\neg\varphi & (3) + (4) + \text{MP} \\
(6) & \vdash_L (\Box\psi_1 \wedge \cdots \wedge \Box\psi_n) \to \Box\neg\varphi & \text{Point 2 of Lemma 3.14} \\
 & & + \text{ P.T. } (a \to b) \to ((b \to c) \to (a \to c)) \\
 & & + \text{ Subst.} + 2 * \text{MP}
\end{array}
$$