Definition 3.1. A **modal logic** (or *logic*, for short) L is a set of modal formulas that

1. contains all propositional tautologies;
2. is closed under *modus ponens (MP)*, i.e.

$$\varphi \in L \text{ and } \varphi \to \psi \in L \text{ implies } \psi \in L;$$

3. and is closed under *uniform substitution*, i.e. if $\varphi \in L$ and ψ can be obtained from φ by uniformly replacing propositional letters in φ with arbitrary formulas;

If $\varphi \in L$, we say that φ is a *theorem* of L and write $\vdash_L \varphi$.

Definition 3.3. A modal logic L is **normal** if it contains the formulas

(K) $\quad \Box(p \to q) \to (\Box p \to \Box q)$

(Dual) $\Diamond p \leftrightarrow \neg\Box\neg p$.

and is closed under *generalization*, i.e. $\vdash_L \varphi$ implies $\vdash_L \Box\varphi$.

Definition 3.7. Let L be a modal logic. Then

- a formula φ is **L-consistent** if $\nvdash_L \neg\varphi$, and *L-inconsistent* otherwise;
- a set of formulas Γ is **L-consistent** if, for every (finite) subset $\{\psi_1, \ldots, \psi_n\} \subseteq \Gamma$, $\psi_1 \wedge \cdots \wedge \psi_n$ is L-consistent; otherwise, Γ is called *L-inconsistent*.