Reminder: Semantics of Modal Logics

A *model* for a set of indices I is a tuple

$$\mathcal{M} = (W, (R_i)_{i \in I}, V)$$

where

- W is a non-empty set of *worlds*;
- $R_i \subseteq W \times W$ for each $i \in I$;
- the *valuation* V assigns to each $p \in \Phi$ a subset $V(p) \subseteq W$.

The component $(W, (R_i)_{i \in I})$ is called a *frame*.

Definition of satisfaction:

$$\begin{array}{rlll}
\mathcal{M}, w \models \top & \text{and} & \mathcal{M}, w \not\models \bot & \\
\mathcal{M}, w \models p & \text{iff} & w \in V(p) & (\text{for } p \in \Phi) \\
\mathcal{M}, w \models \neg\varphi & \text{iff} & \textit{not } \mathcal{M}, w \models \varphi & \\
\mathcal{M}, w \models \varphi \wedge \psi & \text{iff} & \mathcal{M}, w \models \varphi \text{ and } \mathcal{M}, w \models \psi & \\
\mathcal{M}, w \models \Diamond_i \varphi & \text{iff} & (w, w') \in R_i \text{ and } \mathcal{M}, w' \models \varphi \text{ for some } w' \in W &
\end{array}$$

Thus:

$$\mathcal{M}, w \models \Box_i \varphi \quad \text{iff} \quad (w, w') \in R_i \text{ implies } \mathcal{M}, w' \models \varphi \text{ for all } w' \in W$$

If $\mathcal{M}, w \models \varphi$, we say that φ is *satisfied* or *true* in $\mathcal{M}$ at w.