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10. Exercises for the lecture **Term Rewriting Systems**

Exercise 1:

Let R_{Ack} be the (terminating) TRS from the lecture with

$$R_{Ack} = \{a(0,y) \longrightarrow s(y),\ a(s(x),0) \longrightarrow a(x,s(0)),\ a(s(x),s(y)) \longrightarrow a(x,a(s(x),y))\}.$$

Prove that there is no primitiv-recursive function that provides an upper bound for the length of reductions, given the length of the first term. More precisely, prove that there is no primitiv-recursive function f such that, if $t_1 \underset{R_{Ack}}{\longrightarrow} t_2 \underset{R_{Ack}}{\longrightarrow} \dots \underset{R_{Ack}}{\longrightarrow} t_k$, then $k \leq f(|t_1|)$.
Hint: Use Exercise 9.3 and the fact that the Ackermann function grows faster than any primitiv-recursive function.

Exercise 2:

For each of the following pairs (s,t) of terms, check whether $s \trianglerighteq_{emb} t$.
(a) $f(x) \trianglerighteq_{emb} a$? (b) $f(b) \trianglerighteq_{emb} a$? (c) $g(g(x,y),g(a,f(z))) \trianglerighteq_{emb} g(y,g(a,z))$?

Exercise 3:

Prove the following claims:

(a) The reduction relation $\xrightarrow{*}_{emb}$ given by the TRS

$$R_{emb} := \{f(x_1,\dots,x_n) \longrightarrow x_i \mid n \geq 1,\ f \in \Sigma^{(n)} \text{ und } 1 \leq i \leq n\}$$

and the homeomorphic embedding $\trianglerighteq_{emb}$ are identical, i.e., $s \xrightarrow{*}_{emb} t$ iff $s \trianglerighteq_{emb} t$.

(b) $\trianglerighteq_{emb}$ is a partial order.

(c) $\trianglerighteq_{emb}$ is well-founded (prove this last claim without using Kruskall's Theorem).

Exercise 4:

In the proof of Theorem 5.32, we have used that $\trianglerighteq_{emb}$ is a well-partial-order. Explain why $\trianglerighteq_{emb}$ being a well-founded partial order would not have been sufficient.

Exercise 5:

Prove that the TRS $R := \{f(f(x)) \longrightarrow f(g(f(x)))\}$ is terminating.