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11. Exercises for the lecture **Term Rewriting Systems**

Exercise 1:

Prove the first part of Theorem 5.38 of the lecture:
Theorem 5.38.1 Let Σ be a finite signature, $s, t \in \mathcal{T}(\Sigma, V)$, and $>_{lpo}$ a lexicographic path order. We can decide whether $s >_{lpo} t$ in time polynomial in $|s|$ and $|t|$.
Hint: First, show that the condition

$$s >_{lpo} t_j \text{ for all } j \text{ with } 1 \leq j \leq n$$

in (LPO2) can be replaced with

$$s >_{lpo} t_j \text{ for all } j \text{ with } i \leq j \leq n \text{ and } i \text{ such that } s_1 = t_1, \ldots, s_{i-1} = t_{i-1}, \text{ and } s_i >_{lpo} t_i.$$

Use this modified condition (LPO2) to prove that $s >_{lpo} t?$ can be decided in time $\mathcal{O}(|s| \cdot |t|)$.

Exercise 2:

Prove that termination of the TRS $R = \{f(f(x)) \longrightarrow g(x),\ g(g(x)) \longrightarrow f(x)\}$ cannot be proven using a lexicographic path order.

Exercise 3:

Prove termination of the following TRS R using a lexicographic path order:

$$\begin{aligned} R = \{s(x) + (y + z) &\longrightarrow x + (s(s(y)) * z), \\ s(x_1) + (x_2 + (x_3 + x_4)) &\longrightarrow x_1 + (x_2 + (x_3 + x_4))\}. \end{aligned}$$

Exercise 4:

Let Σ be a finite signature with at least one constant symbol, $>$ a strict partial order on Σ, and $>_{lpo}$ the lexicographic path order induced by $>$. Prove that, if $>$ is a total order on Σ, then $>_{lpo}$ is total on ground terms.

Exercise 5:

Prove the following fact:
If $>$ is a reduction order on $\mathcal{T}(\Sigma, V)$ that is total on ground terms, then $>$ satisfies the sub-term property on ground terms, i.e., for each ground term t and position $p \in \mathcal{P}os(t) \setminus \{\epsilon\}$, we have $t > t|_p$.
Discuss the consequence of this fact.