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14. Exercises for the lecture **Term Rewriting Systems**

Exercise 1:

The partial decision procedure described in the proof of Theorem 7.7 of the lecture is rather inefficient: for the input "$s \approx_E t$?", *all* R_i-normal forms of s and t are computed in the ith iteration of the "Repeat"-loop. Show that the following modification of the procedure still yields a partial decision procedure for the word problem:

- Begin with $s_0 := s$ and $t_0 := t$.
- After the ith "Repeat"-loop, compute *one arbitrary* R_i-normal form s_i of s_{i-1} and *one arbitrary* R_i-normal form t_i of t_{i-1}.
- Output "yes" ($s \approx_E t$) iff there exists an n such that $s_n = t_n$.

Hint: show: (1) for all $n \geq 0$ it holds that $s \approx_E s_n$ and $t \approx_E t_n$; (2) since R_∞ is terminating, there exists an n such that $s_n = s_m$ and $t_n = t_m$ for all $m \geq n$.

Exercise 2:

a) Consider the following set of identities:

$$E := \{f(f(x,y),z) \approx f(x,f(y,z)), \quad f(x,x) \approx x, \quad f(f(x,y),x) \approx x\}$$

Apply the rules of the improved completion procedure to E. Use a strategy that resembles the basic completion procedure, but simplifies rules as follows: upon adding new rules, simplify the old ones by means of L-simplify rule and R-simplify rule.

b) Consider the following set of identities:

$$E := \{x + (y + z) \approx (x + y) + z, \quad f(x) + f(y) \approx f(x + y)\}.$$

Apply the completion procedure described above to the input E and the polynomial order induced by

$$\varphi(f)(x) = x + 1, \quad \varphi(+)(x,y) = xy^2.$$

(Exercise 3: on the back)

Exercise 3:

Consider the following completion procedure for ground term rewriting systems (and also recall Exercise 11.4).

Input	G_0, a finite set of ground identities over Σ, $>$, a reduction order total on the set of ground terms over Σ.
Procedure	Apply the rules L-Simplify-Rule, Delete, and Orient until no rule is applicable any more.
Output	a ground term rewriting system.

Show that this procedure: (a) always terminates,
(b) is fair,
(c) is correct, and
(d) never ends in failure.