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5. Exercises for the lecture **Term Rewriting Systems**

Exercise 1:

The reduction relation $\longrightarrow$ enjoys the *diamond property* if

$$\text{for all } x, y_1, y_2\text{: if } y_1 \longleftarrow x \longrightarrow y_2, \text{ then there is some } z \text{ with } y_1 \longrightarrow z \longleftarrow y_2$$

Prove that, if $\longrightarrow$ enjoys the diamond property, then every element x is either in normal form or does not have a normal form.

Exercise 2:

The reduction relation $\longrightarrow$ enjoys the *strass property* if

$$\text{for all } x, y_1, y_2\text{: if } y_1 \neq y_2 \text{ and } y_1 \longleftarrow x \longrightarrow y_2, \text{ then there is some } z \text{ with } y_1 \longrightarrow z \longleftarrow y_2$$

Prove that, if $\longrightarrow$ enjoys the strass property and an element a has a normal form $\widehat{a}$, then

(a) there is no infinite $\longrightarrow$ chain starting at a and

(b) all reductions from a to $\widehat{a}$ are of the same length.

Exercise 3:

Consider the terms $s = f(x, g(h(k(k(y)), x), z), h(x, y))$ and $t = g(z, h(x, k(k(y))))$. Describe $t|_1$, $t|_{1111}$, $t|_{11111}$, $t[s]_2$, and $t[s|_2]_{21}$.

Exercise 4:

Prove Part 2 of Lemma 3.4 by induction on the length of words denoting positions: if $p \in \mathsf{pos}(s)$ and $q \in \mathsf{pos}(t)$, then

$$\begin{aligned} (s[t]_p)|_{pq} &= t_q \\ (s[t]_p)[r]_{pq} &= s[t[r]_q]_p \end{aligned}$$