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9. Exercises for the lecture **Term Rewriting Systems**

Exercise 1:

A TRS R is called *right-irreducible* if each r with $\ell \approx r \in R$ is irreducible.
Prove or disprove the following claim: if R is right-ground and right-irreducible, then R is terminating.

Exercise 2:

Prove Lemma 5.20 of the lecture:

Lemma 5.20 Let $\mathcal{A}$ be a monotone polynomial interpretation of Σ. Then $f^{\mathcal{A}}$ is a montone function for each $f \in \Sigma$.

Exercise 3:

Let R be a finite term rewriting system. We use $|t|_f$ to denote the number of occurrences of the symbol $f \in \Sigma$ in a term t. Prove the following claim:

There is a natural number k_R such that, for all terms s, t, if $s \xrightarrow[R]{} t$, then $|t|_f \leq k_R(|s|_f + 1)$.

Exercise 4:

The following problem is known as *Hilbert's 10th problem*, and it has been proven to be undecidable:
Given: Two polynomials $P, Q \in \mathbb{N}[X_1, \ldots, X_n]$.
Question: Are there $a_1, \ldots, a_n \in \mathbb{N}$ such that $P(a_1, \ldots, a_n) = Q(a_1, \ldots, a_n)$?

Show that the undecidability of Hilbert's 10th problem implies the undecidability of the following problem (which you know from the lecture):
Given: Two polynomials $P, Q \in \mathbb{N}[X_1, \ldots, X_n]$ and a decidable set $A \subseteq \mathbb{N} \setminus \{0\}$.
Question: Is $P >_A Q$? That is, is $P(a_1, \ldots, a_n) > Q(a_1, \ldots, a_n)$ for all $a_1, \ldots, a_n \in A$?

Exercise 5:

Consider the TRS $R = \{g(x, g(y, z)) \longrightarrow g(g(x, y), z), \quad g(g(x, y), z) \longrightarrow g(y, y)\}$. Use a polynomial interpretation to prove that R terminates. Hint: try $p_g = xy + y^2$.