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4. Exercises for the lecture **Term Rewriting Systems**

Exercise 16:

Prove that, for a strict order $\succ$, its induced multiset order $\succ_{mul}$ is a strict order.

Exercise 17:

Let $\succ$ be a strict, decidable order on a finite set A.

(a) Show that the induced multiset order $\succ_{mul}$ is decidable.

(b) Design a simpler decision procedure for the case when $\succ$ is a total strict order on A.

Exercise 18:

Let $(A, \succ)$ be a strict order. Prove or disprove the following claims (remember that a is the *smallest* element of a set M iff $a \in M$ and, for all $b \neq a$ in M, we have $b \succ a$):

(a) If A has a smallest element, then $\succ$ is well-founded.

(b) If every subset of A has a smallest element, then $\succ$ is well-founded.

(c) If $\succ$ is well-founded, then, for each element $a \in A$, there are only finitely many b with $a \succ b$.

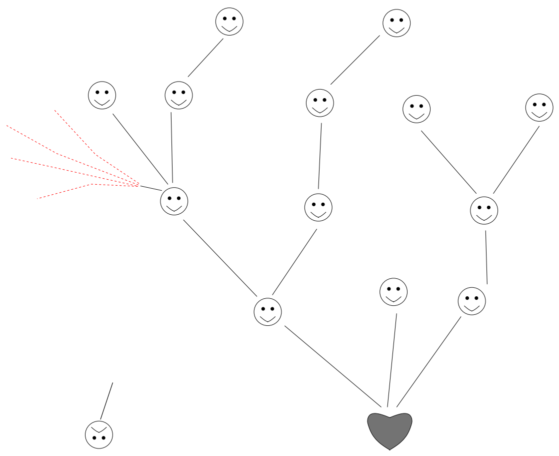
(d) If $\succ$ is well-founded, then, for each element $a \in A$, there is an $n_a \in \mathbb{N}$ such that each $\succ$-path starting at a is of length at most n_a.

(e) If $\succ$ is well-founded, then each subset of A has a smallest element.

Exercise 19:

Consider the following "tree rewrite system", determine whether it is terminating, and describe Herakles' chances to become immortal.
To become immortal, *Herakles* has to solve several tasks, one of which is to kill *Hydra*: Hydra is a strange being, having one heart and a variety of heads (but only finitely many), each of which is connected via a chain of heads to her heart (see the picture below). With one stroke, Herakles can cut off one of Hydra's heads—but Hydra grows, for each head cut off, an arbitrary, finite number of new heads! Each of the newly grown heads is, however, closer to the heart than the one that was cut off. The only way to kill Hydra is to cut off *all* heads.

Herakles strikes:



Hydra with new heads: