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# 12. Exercises for the Course „Logic-based Knowledge Representation“

**Exercise 44:**

Prove Lemma 7.16 from the lecture:

**Lemma 7.16** Let $R_m$ be a reachability relation.

(a) If $R_m$ is symmetric and transitiv, then $R_m$ is euklidian.

(b) The following three conditions are equivalent:

  (i) $R_m$ is symmetric, transitiv, and serial.

  (ii) $R_m$ is reflexiv and euklidian.

  (iii) $R_m$ is an equivalence relation, i.e., reflexiv, transitiv, and symmetric.

**Exercise 45:**

Show that the sets of theorems of $\mathsf{K}_n$, $\mathsf{S4}_n$, and $\mathsf{S5}_n$ are recursively enumerable (in other words, partially decidable).

**Exercise 46:**

Let $\mathcal{K}^{\mathsf{T}_n}$ be the $\mathsf{T}_n$-canonical Kripke structure and let $\Gamma$ be the set of all formulas that are valid in all Kripke structures with reflexive accessibility relations.

Complete the proof of Theorem 7.18.1 by proving the following claim:

The fact that in $\mathcal{K}^{\mathsf{T}_n}$ all accessibility relations are reflexive implies that every formula $\varphi \in \Gamma$ is deducible in $\mathsf{T}_n$. (You are asked to repeat the completeness argument from Theorem 7.11 for $\mathcal{K}^{\mathsf{T}_n}$ assuming without proof Lemma 7.13 and modifying Definition 7.12.)

**Exercise 47:**

A Kripke structure with set of worlds $W$ and a single accessibility relation $R_m$ is *universal* iff $R_m = W \times W$. Let $\Gamma$ be the set of modal formulas valid in all universal Kripke structures.

Prove that $\Gamma = \{\varphi \mid \ \vdash_{\mathsf{S5}_n} \varphi\}$.

**Exercise 48:**

Let $\mathcal{K}$ be a Kripke structure with accessibility relation $R_m$, $\varphi$ a modal logic formula, and $\mathsf{F}$ the axiom schema

$$(\neg[m]\neg\varphi) \Rightarrow [m]\varphi.$$

Prove the following claims:

(a) If $R_m$ is a partial function, then $\mathsf{F}$ holds in $\mathcal{K}$, i.e., $\mathcal{K}, w \models (\neg[m]\neg\varphi) \Rightarrow [m]\varphi$ for all formulae $\varphi$ and all worlds $w$ in $\mathcal{K}$.

(b) If a formula $\psi$ holds in all Kripke structures whose accessibility relation $R_m$ is a partial function, then $\psi$ can be deduced in $\mathsf{K}_n + \mathsf{F}$.