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2. Exercises for the Course „Logic-based Knowledge Representation“

Exercise 6:

Define a generic frame that describes the prototypical object “computer science course”. Use slots

- *Title*,
- *Lecturer*,
- *Type of course*, and
- *Hours per week*.

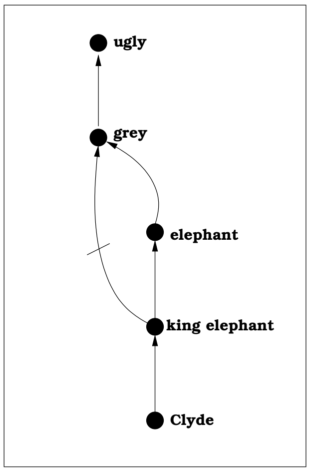
Find other meaningful slots. Then construct an instance frame for the generic frame.

Exercise 7:

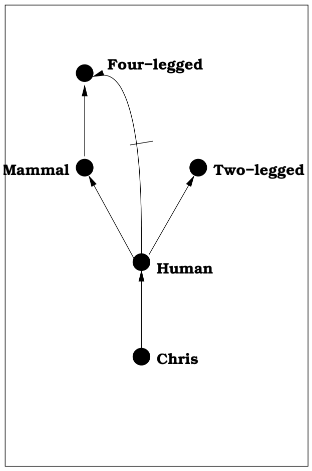
Consider the following non-monotonic inheritance networks and try to resolve the conflicts:

(a) As is well known, king elephants are white rather than gray. Which color has Clyde?

(b) How many legs has Chris?



Network for Part (a)



Network for Part (b)

Exercise 8:

A propositional formula is in *negation normal form* (NNF) if it is built using $\wedge$, $\vee$, and $\neg$ only, and if negation occurs only in front of propositional variables. Prove that for each propositional formula α, there is an equivalent formula β in NNF, which can be efficiently computed from α. What is the size of β in terms of the length of α?

Exercise 9:

Let S be a set of propositional formulae in negation normal form. S contains a *clash* if $\{\alpha, \neg\alpha\} \subseteq S$ for some propositional variable α. If S does not contain a clash, S is *clash-free*. Next, we define the *tableau rules*:

- $\wedge$-rule: If $\beta_1 \wedge \beta_2 \in S$ and not $\{\beta_1, \beta_2\} \subseteq S$, then $S := S \cup \{\beta_1, \beta_2\}$.
- $\vee$-rule: If $\beta_1 \vee \beta_2 \in S$ and $\{\beta_1, \beta_2\} \cap S = \emptyset$, then $S := S \cup \{\beta\}$ for some $\beta \in \{\beta_1, \beta_2\}$.

Please note that the $\vee$-rule is non-deterministic.

(a) Apply the tableau rules exhaustively to

$$\{(p \vee q) \wedge (\neg p \vee \neg q)\}$$

in such a way that it yields a clash-free set of formulae.

(b) Apply the tableau rules exhaustively to

$$\{(p \vee q) \wedge (\neg p \vee \neg q)\}$$

in such a way that it yields set of formulae containing a clash.

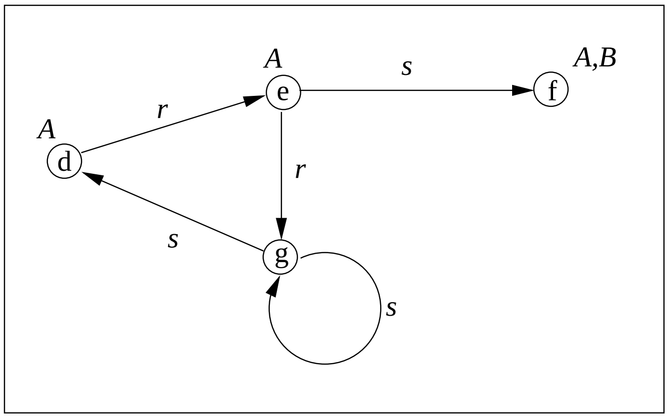
(c) Prove the following proposition: the tableau rules can be applied at most $|S|$ times to a set of propositional formulae, where $|S|$ denotes the sum of the length of all formulae in S.

(d) Prove the following proposition: if the tableau can be applied exhaustively to S in such a way that the resulting set S' is clash-free, then S is satisfiable.

(e) Prove that exhaustive application of tableau rules is a non-deterministic decision procedure for the satisfiability of propositional formulae.

Exercise 10:

Consider the (graphical representation of the) interpretation $\mathcal{I}$ with $\Delta^{\mathcal{I}} = \{d, e, f, g\}$:



For each of the following concepts C, list all elements x of $\Delta^{\mathcal{I}}$ such that $x \in C^{\mathcal{I}}$:

- $A \sqcup B$
- $\exists s.\neg A$
- $\forall s.A$
- $\exists s.\exists s.\exists s.\exists s.A$
- $\forall t.A \sqcap \forall t.\neg A$
- $\neg\exists r.(\neg A \sqcap \neg B)$
- $\exists s.(A \sqcap \forall s.\neg B) \sqcap \neg\forall r.\exists r.(A \sqcup \neg A)$