**Lehrstuhl für Automatentheorie**
Institut für Theoretische Informatik, TU Dresden
Prof. Dr. F. Baader

Hans-Grundig-Str. 25
Dresden Johannstadt
Tel.: 0351/463–38237
Tel.: 0351/463–39171

# 4. Exercises for the Course „Logic-based Knowledge Representation“

**Exercise 16:**

Show that the rules that are used to transform a concept into a concept in negation normal form are equivalence preserving, i.e., show that, for concepts $C, C_1, C_2$, roles $r$, non-negative integers $n$, and positive integers $n'$, the following equivalences hold:

$$\begin{array}{rcl@{\qquad}rcl}
\multicolumn{3}{c}{\neg\neg C \equiv C} & & & \\
\neg(C_1 \sqcup C_2) & \equiv & \neg C_1 \sqcap \neg C_2 & \neg(C_1 \sqcap C_2) & \equiv & \neg C_1 \sqcup \neg C_2 \\
\neg(\forall r.C) & \equiv & \exists r.(\neg C) & \neg(\exists r.C) & \equiv & \forall r.(\neg C) \\
\neg(\leq n\ r) & \equiv & (\geq n+1\ r) & \neg(\geq n'\ r) & \equiv & (\leq n'-1\ r) \\
\multicolumn{3}{c}{\neg(\geq 0\ r) \equiv \bot} & & &
\end{array}$$

**Exercise 17:**

Let A, B, C, and D be concept names and r a role name.

(a) Use the tableau-based algorithm presented in the lecture to decide consistency of the following ABoxes. If an ABox is consistent, describe a canonical model.

(i) $\mathcal{A}_1 = \{\mathsf{a}\colon (\exists \mathsf{r}.(\mathsf{C} \sqcup \mathsf{D}) \sqcap \forall \mathsf{r}.(\neg \mathsf{C} \sqcap \neg \mathsf{D})) \sqcup (\exists \mathsf{r}.\mathsf{D} \sqcap \forall \mathsf{r}.\forall \mathsf{r}.\neg \mathsf{D})\}$

(ii) $\mathcal{A}_2 = \{\mathsf{a}\colon \exists \mathsf{r}.(\leq 1\ \mathsf{r}) \sqcap \exists \mathsf{r}.(\exists \mathsf{r}.\mathsf{A} \sqcap \exists \mathsf{r}.\mathsf{B} \sqcap \forall \mathsf{r}.\mathsf{C}),\ \mathsf{a}\colon \forall \mathsf{r}.((\exists \mathsf{r}.\mathsf{A} \sqcap \exists \mathsf{r}.\neg \mathsf{A}) \sqcup (\exists \mathsf{r}.\mathsf{A} \sqcap \exists \mathsf{r}.\mathsf{B}))\}$

(b) Use the tableau-based algorithm to decide the following subsumption relationship.

$$\exists \mathsf{r}.\mathsf{A} \sqcap \exists \mathsf{r}.\mathsf{B} \sqsubseteq^{?} \exists \mathsf{r}.(\mathsf{A} \sqcap \mathsf{B})$$

**Exercise 18:**

Complete the proof of Lemma 4.6 by showing that, if the application of the

(a) $\sqcap$–Rule,

(b) $\forall$–Rule, or

(c) $\geq$-Rule

to $\mathcal{A}$ yields $\mathcal{A}_1, \ldots, \mathcal{A}_n$, then $\mathcal{A}$ is consistent iff, for some $1 \leq i \leq n$, $\mathcal{A}_i$ is consistent.