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5. Exercises for the Course „Logic-based Knowledge Representation“

Exercise 19:

For the $\geq$-Rule of the tableau-based algorithm to be applicable to some $\mathsf{a}\colon (\geq\ n\ r) \in \mathcal{A}$, the following condition must be satisfied:
there are no n individual names $\mathsf{b}_1, \ldots \mathsf{b}_n$ with

$$\{(\mathsf{a}, \mathsf{b}_1)\colon r, \ldots, (\mathsf{a}, \mathsf{b}_n)\colon r\} \cup \{\mathsf{b}_i \not\doteq \mathsf{b}_j \mid 1 \leq i < j \leq n\} \subseteq \mathcal{A}\,.$$

(a) It is natural to view (and decide) this condition as a graph-theoretical problem by translating inequality assertions into a graph. Describe the graph-theoretic problem corresponding to the above applicability condition. What is the complexity of this problem?

(b) Is it possible to replace the inequality assertions in ABoxes with a more clever data structure, such that checking the applicability of $\geq$-Rule becomes less complex?

Exercise 20:

Let K be a set of constructors to generate complex concepts.

- A constructor $k \in K$ is *redundant in* K, if, for each concept C using constructors in K, there exists an equivalent concept C' using only constructors in $K \setminus \{k\}$.
- Two sets K and K' of constructors are *equivalent* if, for each concept C using only constructors in K, there exists an equivalent concept C' using only constructors in K' and, vice versa, for each concept C using only constructors in K', there exists an equivalent concept C' using only constructors in K.

Define, for each of the following two sets of constructors K, a minimal set K' such that K' and K are equivalent and K' does not contain any redundant constructor.

(a) $K := \{\neg, \sqcup, \sqcap, \forall, \exists\}$,

(b) $K := \{\leq n, \geq n, \neg, \sqcup, \sqcap, \forall, \exists\}$.