Lehrstuhl für Automatentheorie
Institut für Theoretische Informatik, TU Dresden
Prof. Dr. F. Baader

Hans-Grundig-Str. 25
Dresden Johannstadt
Tel.: 0351/463–38237
Tel.: 0351/463–39171

7. Exercises for the Course „Logic-based Knowledge Representation“

Exercise 24:

Recall the following notions:

1. A *polynomial reduction* from a problem $P \subseteq X$ to a problem $P' \subseteq X'$ is a polynomial time computable function σ that takes every $x \in X$ to a $\sigma(x) \in X'$ such that $x \in P$ iff $\sigma(x) \in P'$.
2. A problem P is PSPACE-*hard* iff, for every problem P' that is *in* PSPACE, there exists a polynomial reduction σ from P' to P.

Prove the following claims:

(a) If a problem P_1 is PSPACE-hard and there exists a polynomial reduction from P_1 to another problem P_2, then P_2 is PSPACE-hard.

(b) If a problem P_1 is undecidable and there exists a polynomial reduction from P_1 to another problem P_2, then P_2 is undecidable.

Exercise 25:

To extend the $\mathcal{ALCN}$-tableau algorithm from the lecture such that it can, additionally, deal with role value maps, one could think of adding the following two rules:

Replace $\mathcal{A} \in M$ by $\mathcal{A}'$ iff the following conditions hold:

1. $a : (R \sqsubseteq S) \in \mathcal{A}$, where $R = r_1 \cdots r_n$ and $S = s_1 \cdots s_m$
2. $\{(a, b_1) : r_1, (b_1, b_2) : r_2, \ldots, (b_{n-1}, b_n) : r_n\} \subseteq \mathcal{A}$
3. there exist no $c_1, \ldots, c_m$ such that $c_m = b_n$ and $\{(a, c_1) : s_1, (c_1, c_2) : s_2, \ldots, (c_{m-1}, c_m) : s_m\} \subseteq \mathcal{A}$
4. $\mathcal{A}' = \mathcal{A} \cup \{(a, c_1) : s_1, (c_1, c_2) : s_2, \ldots, (c_{m-1}, c_m) : s_m\}$ where $c_1, \ldots, c_{m-1}$ are new individuals and $c_m = b_n$.

Then do the following:

(a) Apply the extended tableau algorithm to the ABox

$$\{a : (\exists r.\exists s.\exists s.A) \sqcap (\forall r.\neg A) \sqcap (rs \sqsubseteq r)\}$$

and decide whether it is consistent.

(b) Devise an ABox on which the extended algorithm does not terminate.

Exercise 26:

We define a slightly different variant of the domino problem: A solution of a domino system $\mathcal{D} = (D, H, V)$ (as defined in the lecture) *w.r.t. an initial condition* $a = a_0, \ldots, a_n$ is a mapping $\tau : \mathbb{N} \times \mathbb{N} \to D$ such that

- $\tau(x, y) = d$ and $\tau(x + 1, y) = d'$ implies $(d, d') \in H$,
- $\tau(x, y) = d$ and $\tau(x, y + 1) = d'$ implies $(d, d') \in V$, and
- $\tau(i, 0) = a_i$ for $i \leq n$.

Prove that it is undecidable whether a domino system $\mathcal{D}$ has a solution w.r.t. an initial condition a.

Hints:

(a) Reduce the halting problem of Turing machines (TMs) to this problem. Consider only TMs which use one-side infinite tapes and are started on the empty tape (for this class of TMs, the halting problem is already undecidable).

(b) Use the first row $\{(i, 0) \mid i \in \mathbb{N}\}$ of the $\mathbb{N} \times \mathbb{N}$ structure to represent the initial tape content, the second row $\{(i, 1) \mid i \in \mathbb{N}\}$ to represent the tape content after the first step of the TM, and so forth.

(c) Define additional tile types that, apart from determining the content of a tape cell, represent the position of the TM's head and its current state.

(d) Use the initial condition to ensure that the TM starts in the initial state and that the tape is initially empty.

(e) Be careful: the head's position and state must be unique for each step of the computation.