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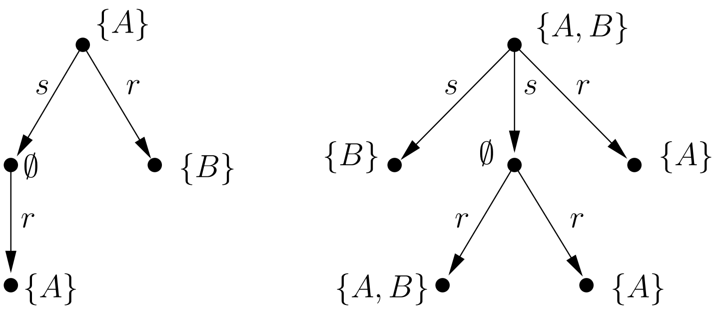
10. Exercises for the Course „Logic-based Knowledge Representation"

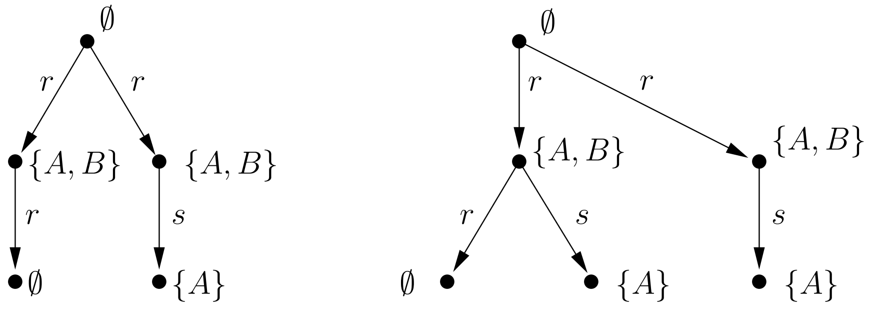
Exercise 36:

Compute all possible homomorphisms between the following description trees:

1.



2.



Exercise 37:

Transform the following $\mathcal{EL}$-TBox into a general TBox in normal form:
$\mathcal{T} = \{C \sqcap \exists r.(D_1 \sqcap D_2) \sqsubseteq \exists s.(E_1 \sqcap E_2) \sqcap \exists s.(E_1 \sqcap E_3)\}$

Exercise 38:

Prove claim from the lecture:
The application of the rules (NF1, NF2, NF3, NF4, NF5) to a $\mathcal{EL}$-TBox $\mathcal{T}$ terminates in polynomial time and the obtained general TBox $\hat{\mathcal{T}}$ is normalized and satisfies:
$(*)$ $\quad A \sqsubseteq_{\mathcal{T}} B$ iff $A \sqsubseteq_{\hat{\mathcal{T}}} B$

Exercise 39:

Let $\mathcal{L}$ be a Description Logic and C, D, and E be $\mathcal{L}$-concept terms. Prove that the following properties hold:

(a) $\operatorname{lcs}(C, D) = \operatorname{lcs}(D, C)$

(b) $\operatorname{lcs}(C, \operatorname{lcs}(D, E)) = \operatorname{lcs}(\operatorname{lcs}(C, D), E)$

(c) The least common subsumer of C and D is unique up to equivalence.

Exercise 40:

Compute the least common subsumer of

(a) the following pairs of $\mathcal{FL}_0$-concept terms:

$$\begin{aligned} C &= A_1 \sqcap A_2 \sqcap \forall r.(A_1 \sqcap \forall s.A_1 \sqcap \forall r.A_2) \sqcap \forall r.\forall s.\forall r.(A_1 \sqcap A_2 \sqcap \forall s.A_2) \\ D &= A_2 \sqcap \forall r.\forall r.(A_2 \sqcap A_3) \sqcap \forall s.A_2 \sqcap \forall r.(A_2 \sqcap \forall s.(A_2 \sqcap \forall r.(A_2 \sqcap A_3))) \end{aligned}$$

(b) the following pairs of $\mathcal{EL}$-concept terms:

$$\begin{aligned} C &= A \sqcap \exists r.(A \sqcap \exists (A \sqcap B)) \sqcap \exists r.(B \sqcap \exists s.B) \sqcap \exists s.(A \sqcap \exists r.\top) \\ D &= B \sqcap \exists r.(A \sqcap B \sqcap \exists s.(A \sqcap B \sqcap \exists s.(A \sqcap B))) \sqcap \exists s.(A \sqcap B) \sqcap \exists s.(B \sqcap \exists r.\top) \end{aligned}$$

Exercise 41:

(a) Consider the following $\mathcal{EL}$-concept terms:

$$\begin{aligned} C_1 &= \exists r.(A \sqcap \exists r.(A \sqcap B \sqcap \exists r.(A \sqcap B))) \sqcap \exists r.(B \sqcap \exists r.(A \sqcap B \sqcap \exists r.(A \sqcap B))) \\ C_2 &= \exists r.(A \sqcap B \sqcap \exists r.(A \sqcap \exists r.(A \sqcap B)) \sqcap \exists r.(B \sqcap \exists r.(A \sqcap B))) \\ C_3 &= \exists r.(A \sqcap B \sqcap \exists r.(A \sqcap B \sqcap \exists r.A \sqcap \exists r.B)) \end{aligned}$$

Compute $D = \operatorname{lcs}(C_1, \operatorname{lcs}(C_2, C_3))$ and prove that there is no $\mathcal{EL}$-concept term E such that D and E are equivalent and E's length is smaller than $2^4 - 1$ (i.e., the number of nodes in a tree of depth 3).

(b) Generalize this example to arbitrary number of concepts, i.e. show that for an arbitrary number n, there are concepts $C_1, \ldots, C_n$, such that $\operatorname{lcs}(C_1, (\operatorname{lcs}(C_2, \operatorname{lcs}(\ldots, \operatorname{lcs}(C_{n-1}, C_n) \ldots)$ is of length $2^{n+1} - 1$ and there is no smaller concept term equivalent to it.