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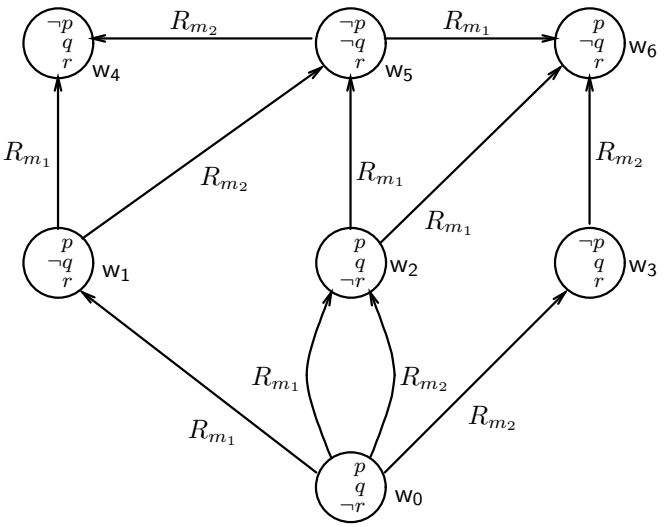
11. Exercises for the Course „Logic-based Knowledge Representation“

Exercise 42:

The Kripke structure $\mathcal{K} := (\{\mathsf{w}_0, \ldots, \mathsf{w}_6\}, \pi, R)$ is given by the following graph:



(a) Prove resp. disprove the following statements:

(i) $\mathcal{K}, \mathsf{w}_0 \models (r \to p) \wedge [m_1]\langle m_1\rangle(p \to q)$

(ii) $\mathcal{K}, \mathsf{w}_1 \models \Big([m_2]\langle m_1\rangle r \wedge \langle m_2\rangle[m_1]r\Big) \to \Big([m_2]\langle m_2\rangle q \vee [m_1]\langle m_1\rangle\neg q\Big)$

(iii) $\mathcal{K}, \mathsf{w}_2 \models [m_1](p \to r) \wedge [m_2](p \wedge \neg p)$

(iv) $\mathcal{K}, \mathsf{w}_3 \models [m_1](p \vee q) \wedge \langle m_2\rangle(p \to r) \wedge [m_2]\langle m_1\rangle\neg q\,.$

(b) Give, for each formula in (a), the corresponding $\mathcal{ALC}$ concept term.

(c) Describe an interpretation $\mathcal{I}$ that corresponds one-to-one to the Kripke structure $\mathcal{K}$ (in the sense mentioned in the lecture).

Exercise 43:

For $n \in \mathbb{N} \setminus \{0\}$, let $M_n := \{m_1, \ldots, m_n\}$ be a set of modal parameters and $m \in M_n$. Prove resp. disprove the validity of the following K_n-formulae:

(a) $[m]p \to p$ (b) $[m]p \to [m][m]p$
(c) $p \to [m]\langle m\rangle p$ (d) $\langle m\rangle p \to [m]\langle m\rangle p$
(e) $([m](p \to q)) \to ([m]p \to [m]q)\,.$

Exercise 44:

In the lecture, the filtration technique was used to prove that K_n has the finite model property. Use the technique to prove that $\mathsf{K4}_n$ has the finite model property, where $\mathsf{K4}_n$ is syntactically the same logic as K_n, but the semantics is different: we only consider those Kripke structures where each accessibility relation is transitive.

Exercise 45:

Using axioms and inference rules give proofs for the following K_n-formulae:

(a) $[n]([m](p \vee q) \vee \neg[m](p \vee q))$

(b) $[m](p \wedge q) \to [m]q$ and $[m](p \wedge q) \to [m]p$

(c) $[m](p \wedge q) \to ([m]p \wedge [m]q)$

Exercise 46:

Let Δ and Γ be sets of K_n-formulae. Prove or refute the following claims:

(a) if Γ is consistent and $\Delta \subseteq \Gamma$, then Δ is consistent

(b) if Γ is inconsistent and $\Gamma \subseteq \Delta$, then Δ is inconsistent.

(c) Let $\Gamma = \bigcup_{i \geq 0} \Gamma_i$. Then Γ is consistent iff all Γ_i are consistent.

(d) Let $\Gamma = \bigcup_{i \geq 0} \Gamma_i$ and $\Gamma_0 \subseteq \Gamma_1 \subseteq \Gamma_2 \cdots$. Then Γ is consistent iff all Γ_i are consistent.