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5. Exercises for the Course „Logic-based Knowledge Representation“

Exercise 17:

Prove that the procedure for expanding TBoxes of Theorem 3.6 always terminates, and that it returns a TBox that is equivalent to its input.

Hint: There are many ways to prove this claim. You can try one of these:

(a) Define an order on concepts in a TBox, extend this to the multiset order (the right sides of the definitions in the TBox can be treated as multisets of concepts), and with the help of this define a measure on the TBox which is decreased with every step of the expansion.

(b) Construct a set of trees for a TBox T, i.e. for each concept defined in T, construct a tree drawing an edge from this concept(root) to each concept in its definition, and then add additional edges from these concepts to the concepts in their definitions, etc. What is decreasing with an exapansion step?

(c) Count for each concept defined in the TBox the number of concept names (directly or indirectly) used in the definition of this concept.

Exercise 18:

Which of the following statements are valid? Please give reasons for your answers.

$$\begin{array}{lrcl}
(a) & \forall \mathsf{r}.(\mathsf{C} \sqcap \mathsf{D}) & \sqsubseteq & \forall \mathsf{r}.\mathsf{C} \sqcap \forall \mathsf{r}.\mathsf{D} \\
(b) & \forall \mathsf{r}.\mathsf{C} \sqcap \forall \mathsf{r}.\mathsf{D} & \sqsubseteq & \forall \mathsf{r}.(\mathsf{C} \sqcap \mathsf{D}) \\
(c) & \forall \mathsf{r}.(\mathsf{C} \sqcup \mathsf{D}) & \sqsubseteq & \forall \mathsf{r}.\mathsf{C} \sqcup \forall \mathsf{r}.\mathsf{D} \\
(d) & \forall \mathsf{r}.\mathsf{C} \sqcup \forall \mathsf{r}.\mathsf{D} & \sqsubseteq & \forall \mathsf{r}.(\mathsf{C} \sqcup \mathsf{D}) \\
(e) & \exists \mathsf{r}.(\mathsf{C} \sqcap \mathsf{D}) & \sqsubseteq & \exists \mathsf{r}.\mathsf{C} \sqcap \exists \mathsf{r}.\mathsf{D} \\
(f) & \exists \mathsf{r}.\mathsf{C} \sqcap \exists \mathsf{r}.\mathsf{D} & \sqsubseteq & \exists \mathsf{r}.(\mathsf{C} \sqcap \mathsf{D}) \\
(g) & \exists \mathsf{r}.\mathsf{C} \sqcup \exists \mathsf{r}.\mathsf{D} & \sqsubseteq & \exists \mathsf{r}.(\mathsf{C} \sqcup \mathsf{D}) \\
(h) & \exists \mathsf{r}.(\mathsf{C} \sqcup \mathsf{D}) & \sqsubseteq & \exists \mathsf{r}.\mathsf{C} \sqcup \exists \mathsf{r}.\mathsf{D}
\end{array}$$

Exercise 19:

Show that the following rules that are used to transform a concept into a concept in negation normal form are equivalence preserving, i.e., show that, for all concepts C, C_1, C_2, roles r, non-negative integers n, and positive integers n', the right side of each rule is equivalent to the left side:

$$\begin{aligned}
&\neg\neg C \Longrightarrow C\\
&\neg(C_1 \sqcup C_2) \Longrightarrow \neg C_1 \sqcap \neg C_2\\
&\neg(C_1 \sqcap C_2) \Longrightarrow \neg C_1 \sqcup \neg C_2\\
&\neg(\forall r.C) \Longrightarrow \exists r.(\neg C)\\
&\neg(\exists r.C) \Longrightarrow \forall r.(\neg C)\\
&\neg(\leq n\ r) \Longrightarrow (\geq n+1\ r)\\
&\neg(\geq n'\ r) \Rightarrow (\leq n'-1\ r)\\
&\neg(\geq 0\ r) \Longrightarrow \bot\\
&\neg\top \Longrightarrow \bot\\
&\neg\bot \Longrightarrow \top
\end{aligned}$$

Show that the exhaustive application of these rules to a concept always terminates with the concept in negation normal form.

Exercise 20:

Let A, B, C, and D be concept names and r a role name.

(a) Use the tableau-based algorithm presented in the lecture to decide consistency of the following ABoxes. If an ABox is consistent, describe a canonical model.

(i) $\mathcal{A}_1 = \{\mathsf{a} \colon (\exists \mathsf{r}.(\mathsf{C} \sqcup \mathsf{D}) \ \sqcap\ \forall \mathsf{r}.(\neg \mathsf{C} \sqcap \neg \mathsf{D}))\ \sqcup\ (\exists \mathsf{r}.\mathsf{D}\ \sqcap\ \forall \mathsf{r}.\forall \mathsf{r}.\neg \mathsf{D})\}$

(ii) $\mathcal{A}_2 = \{\mathsf{a} \colon \exists \mathsf{r}.(\leq 1\ \mathsf{r}) \sqcap \exists \mathsf{r}.(\exists \mathsf{r}.\mathsf{A} \sqcap \exists \mathsf{r}.\mathsf{B} \sqcap \forall \mathsf{r}.\mathsf{C}),\ \mathsf{a} \colon \forall \mathsf{r}.((\exists \mathsf{r}.\mathsf{A} \sqcap \exists \mathsf{r}.\neg \mathsf{A}) \sqcup (\exists \mathsf{r}.\mathsf{A} \sqcap \exists \mathsf{r}.\mathsf{B}))\}$

(b) Use the tableau-based algorithm to decide the following subsumption relationship.

$$\exists \mathsf{r}.\mathsf{A} \sqcap \exists \mathsf{r}.\mathsf{B} \ \sqsubseteq^{?}\ \exists \mathsf{r}.(\mathsf{A} \sqcap \mathsf{B})$$

Exercise 21:

Complete the proof of Lemma 4.6 by showing that, if the application of the

(a) $\sqcap$–Rule,

(b) $\forall$–Rule, or

(c) $\geq$-Rule

to $\mathcal{A}$ yields $\mathcal{A}_1, \ldots, \mathcal{A}_n$, then $\mathcal{A}$ is consistent iff, for some $1 \leq i \leq n$, $\mathcal{A}_i$ is consistent.