2. Exercises for the Course „Description Logics“

Exercise 6:
Let $G = (V, E)$ be a directed graph represented as a set of PROLOG facts

$$\{ \text{directly-connected}(x, y). \mid (x, y) \in E \}.$$ 

Consider three PROLOG programs that compute whether two nodes of a graph are connected:

(a) \(\text{connected}(x, y) : \text{directly-connected}(x, y)\)
    \(\quad \text{connected}(x, y) : \text{directly-connected}(x, z), \text{connected}(z, y)\)

(b) \(\text{connected}(x, y) : \text{directly-connected}(x, y)\)
    \(\quad \text{connected}(x, y) : \text{connected}(z, y), \text{directly-connected}(x, z)\)

(c) \(\text{connected}(x, y) : \text{directly-connected}(x, z), \text{connected}(z, y)\)
    \(\quad \text{connected}(x, y) : \text{directly-connected}(x, y)\)

Do the following:
- For each of the three programs, determine whether it is sound, complete, and terminating.
- Rewrite each program as a set of implication in first-order logic. Are the three sets logically equivalent?
- A KR formalism is declarative if the meaning of its terms is defined independently of a concrete interpreter or reasoning algorithm. Is KR in PROLOG declarative?

Exercise 7:
Let $\alpha$ and $\beta$ be propositional formulae. Prove or disprove the following propositions:

(a) If $\varphi \rightarrow \psi$ and $\varphi$ are valid, then $\psi$ is valid.
(b) If $\varphi \rightarrow \psi$ and $\varphi$ are satisfiable, then $\psi$ is satisfiable.
(c) If $\varphi \rightarrow \psi$ is valid and $\varphi$ is satisfiable, then $\psi$ is satisfiable.

Exercise 8:
A propositional formula using only the constructors $\land$, $\lor$, and $\neg$ is in negation normal form (NNF) if negation occurs only in front of propositional variables.

Prove that each propositional formula can be transformed into an equivalent one in NNF.

Exercise 9:
Define a generic frame that describes the prototypical object “computer science course”. Use slots

- Title,
- Lecturer,
- Type of course, and
- Hours per week.

Find other meaningful slots. Then construct an instance frame for the generic frame.