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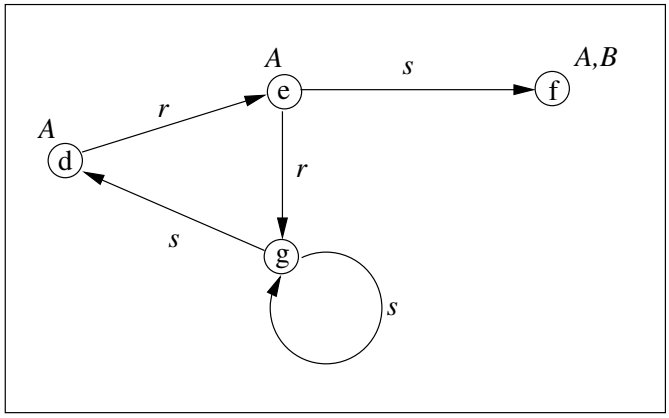
3. Exercises for the Course „Description Logics“

Exercise 10:

Consider the (graphical representation of the) interpretation $\mathcal{I}$ with $\Delta^{\mathcal{I}} = \{d, e, f, g\}$:



For each of the following $\mathcal{ALC}$-concepts C, list all elements x of $\Delta^{\mathcal{I}}$ such that $x \in C^{\mathcal{I}}$:

- $A \sqcup B$
- $\exists s.\neg A$
- $\forall s.A$
- $\exists s.\exists s.\exists s.\exists s.A$
- $\forall t.A \sqcap \forall t.\neg A$
- $\neg\exists r.(\neg A \sqcap \neg B)$
- $\exists s.(A \sqcap \forall S.\neg B) \sqcap \neg\forall r.\exists r.(A \sqcup \neg A)$

Exercise 11:

Construct a TBox describing a supermarket. Use concept names such as Supermarket, Shop, Food, Employee, and role names such as sells and works-for.

Exercise 12:

Prove or disprove the following (for the description logic $\mathcal{ALC}$):

- There is a TBox that has no models at all.
- There is a TBox that has only finite models.
- Every TBox has either no models at all or infinitely many models.
- There is a TBox $\mathcal{T}$ such that all models of $\mathcal{T}$ are either infinite or contain a cycle (when viewed as a graph).
- For every TBox $\mathcal{T}$, there is an equivalent TBox $\mathcal{T}'$ that contains only a single GCI (where two TBoxes are *equivalent* if they have the same models).

Exercise 13:

Let $\mathcal{A}$ be the ABox consisting of the following assertions:

$$\begin{array}{cc} (\mathsf{Ralf},\mathsf{Claudia}) : \mathsf{likes} & (\mathsf{Ralf},\mathsf{Jörg}) : \mathsf{likes} \\ (\mathsf{Claudia},\mathsf{Jörg}) : \mathsf{is\text{-}neighbour\text{-}of} & (\mathsf{Jörg},\mathsf{Andrea}) : \mathsf{is\text{-}neighbour\text{-}of} \\ \mathsf{Claudia} : \mathsf{blond} & \mathsf{Andrea} : \neg\mathsf{blond} \end{array}$$

(a) Does $\mathcal{A}$ have a model?

(b) Is Ralf an instance of the concept $\exists \mathsf{likes}.(\mathsf{blond} \sqcap \exists \mathsf{is\text{-}neighbour\text{-}of}.\neg\mathsf{blond})$ in all models of $\mathcal{A}$?

(c) Is Ralf an instance of the concept $\exists \mathsf{likes}.(\exists \mathsf{is\text{-}neighbour\text{-}of}.\ (\forall \mathsf{is\text{-}neighbour\text{-}of}.\ \neg\mathsf{blond}))$ in all models of $\mathcal{A}$?

Exercise 14:

Extend the TBox from Exercise 11 to a knowledge base by constructing an appropriate ABox.

Exercise 15:

Prove or disprove the following (for the description logic $\mathcal{ALC}$):

- There is an ABox that has no models at all.
- There is an ABox that has only finite models.
- Every ABox has either no model or infinitely many models.
- There is an ABox $\mathcal{A}$ such that all models of $\mathcal{A}$ contain a cycle (when viewed as a graph).

Exercise 16:

Let the interpretation $\mathcal{I}$ be obtained from the interpretation $\mathcal{I}'$ from Example 2.6 given in the lecture by the following modifications:

- h, m become instances of $\mathsf{Student}$;
- $u6$, $g4$ are no longer instances of Person.

Prove that $\mathcal{I}'$ is a model of the knowledge base $= (,)$, where is the TBox from Example 2.4, and the ABox from Example 2.6.