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4. Exercises for the Course „Description Logics“

Exercise 17:

Revisit the procedure for expanding TBoxes given in the proof of Proposition 2.7. Prove that

(a) this procedure always terminates, and

(b) that it returns a TBox that is equivalent to its input.

Let an *extended* acyclic TBox be an acylic TBox that also admits primitive definitions, with the definitions of unambiguity and acyclicity extended to take into account also primitive definitions. Does the expansion procedure produce equivalent TBoxes when applied to extended acyclic TBoxes?

Hint for proving termination: count, for each concept name A, the number of concept names (directly or indirectly) used in the definition of A.

Exercise 18:

Let $\mathcal{T}$ be an acyclic TBox. Prove without using TBox expansion that every primitive interpretation $\mathcal{I}$ can be extended to a model of $\mathcal{T}$ in a unique way.

Exercise 19:

Let $\mathcal{K} = (\mathcal{T}, \mathcal{A})$. We write $C \sqsubseteq_{\mathcal{K}} D$ if $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ for all models $\mathcal{I}$ of $\mathcal{K}$. Prove that for all $\mathcal{ALC}$-concepts C and D, we have $C \sqsubseteq_{\mathcal{K}} D$ iff $C \sqsubseteq_{\mathcal{T}} D$.

Exercise 20:

Which of the following statements are true? Give reasons.

$$
\begin{array}{llclllcl}
(a) & \forall r.(A \sqcap B) & \sqsubseteq & \forall r.A \sqcap \forall r.B & (e) & \exists r.(A \sqcap B) & \sqsubseteq & \exists r.A \sqcap \exists r.B \\
(b) & \forall r.A \sqcap \forall r.B & \sqsubseteq & \forall r.(A \sqcap B) & (f) & \exists r.A \sqcap \exists r.B & \sqsubseteq & \exists r.(A \sqcap B) \\
(c) & \forall r.(A \sqcup B) & \sqsubseteq & \forall r.A \sqcup \forall r.B & (h) & \exists r.(A \sqcup B) & \sqsubseteq & \exists r.A \sqcup \exists r.B \\
(d) & \forall r.A \sqcup \forall r.B & \sqsubseteq & \forall r.(A \sqcup B) & (g) & \exists r.A \sqcup \exists r.B & \sqsubseteq & \exists r.(A \sqcup B)
\end{array}
$$

Exercise 21:

For each $i > 0$, give

(a) a satisfiable $\mathcal{ALC}$-concept C_i that has only models with at least i elements;

(b) a satisfiable $\mathcal{ALC}$-concept D_i that has only models with at least 2^i elements.

For Part (b), construct the concepts $D_1, D_2, \ldots$ such that the length of D_i bounded by $p(i)$, for p a polynomial.