Lehrstuhl für Automatentheorie
Institut für Theoretische Informatik, TU Dresden
Prof. Dr. F. Baader

Nöthnitzer Str. 46
01187 Dresden
Tel.: 0351/463–38237
Tel.: 0351/463–39171

5. Exercises for the Course „Description Logics“

Exercise 22:

Let C and D be $\mathcal{ALC}$-concepts that use only the constructors $\neg$, $\sqcap$, and $\sqcup$, but not $\exists r.C$ and $\forall r.C$. With C, we can associate a propositional formula φ_C by replacing each concept name A with a propositional variable p_A, "$\sqcup$" with "$\vee$", and "$\sqcap$" with "$\wedge$". Similarly for D and φ_D. Prove that $C \sqsubseteq D$ iff $\varphi_C \rightarrow \varphi_D$ is valid in propositional logic.

Exercise 23:

Consider the following combinations of concepts C and TBoxes $\mathcal{T}$. Determine whether C is satisfiable w.r.t. $\mathcal{T}$:

- $C = A \sqcap B$, $\mathcal{T} = \{A \sqsubseteq \exists r.X \sqcap \exists r.\neg X,\ A \sqsubseteq \forall r.Y,\ B \equiv (\leq 1\, r.Y)\}$
- $C = A \sqcap B$, $\mathcal{T} = \{A \sqsubseteq (\geq 5\, r.A) \sqcap (\leq 2\, r.X),\ B \sqsubseteq (\leq 2\, r.\neg X)\}$
- $C = A \sqcap \neg X$, $\mathcal{T} = \{A \sqsubseteq \exists r.B,\ B \sqsubseteq \exists r^-.X,\ X \sqsubseteq \forall r.(\leq 1\, r^-.\top)\}$

Exercise 24:

Prove that the concept $\forall r^-.\bot$ cannot be expressed in $\mathcal{ALC}$, i.e., that there is no $\mathcal{ALC}$-concept that is equivalent to $\forall r^-.\bot$.

Exercise 25:

Prove that the concept $(\leq 1\, r.\top)$ cannot be expressed in $\mathcal{ALC}$.

Proceed as follows: assume that C is an $\mathcal{ALC}$-concept equivalent to $(\leq 1\, r.\top)$, and take a model $\mathcal{I}$ of C.[1] Construct a model $\mathcal{I}_\omega$ as follows:

- $\Delta^{\mathcal{I}_\omega} := \Delta^{\mathcal{I}} \times \mathbb{N}$;
- $A^{\mathcal{I}_\omega} := \{\langle d, i\rangle \mid d \in A^{\mathcal{I}} \text{ and } i \geq 0\}$;
- $r^{\mathcal{I}_\omega} := \{(\langle d, i\rangle, \langle d', i'\rangle) \mid (d, d') \in r^{\mathcal{I}} \text{ and } i, i' \geq 0\}$.

Prove that $\mathcal{I}_\omega$ is a model of C and conclude that C is not equivalent to $(\leq 1\, r.\top)$. Can $(\leq 1\, n.\top)$ be expressed in $\mathcal{ALC}$, for any $n \geq 0$?

Exercise 26:

Assume we want to define the concept of those students that attend *all* CL courses, using concept names such as Student and CL-Course and role names such as attends. Can this be expressed in $\mathcal{ALC}$? If not, propose a new concept-forming constructor that allows to express it. Define its syntax and semantics.

[1] We call $\mathcal{I}$ a *model* of C if $C^{\mathcal{I}} \neq \emptyset$.