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6. Exercises for the Course „Description Logics“

Exercise 27:

Prove the following Points of Theorem 2.13:

(a) C is satisfiable w.r.t. $\mathcal{T}$ iff $C \not\sqsubseteq_{\mathcal{T}} \bot$;

(b) C is satisfiable w.r.t. $\mathcal{T}$ iff $(\mathcal{T}, \{C(a)\})$ is consistent;

(c) a is an instance of C w.r.t. $\mathcal{K}$ iff $(\mathcal{T}, \mathcal{A} \cup \{\neg C(a)\})$ is inconsistent.

Exercise 28:

Recall that the description logic $\mathcal{ALC}$ is equipped with the concept constructors negation ($\neg$), conjunction ($\sqcap$), disjunction ($\sqcup$), existential restriction ($\exists r.C$), and universal restriction ($\forall r.C$). Each subset of this set of constructors gives rise to a fragment of $\mathcal{ALC}$.

Identify all minimal fragments that are equivalent to $\mathcal{ALC}$ in the sense that for every $\mathcal{ALC}$-concept, there is an equivalent concept in the fragment.

Exercise 29:

Let $\mathcal{T}$ be a TBox, and $\mathsf{N}_\mathsf{C}^\mathcal{T}$ denote the set of all concept names in $\mathcal{T}$. Let $\approx_\mathcal{T}$ be the relation on $\mathsf{N}_\mathsf{C}^\mathcal{T}$ defined by setting $A \approx_\mathcal{T} B$ iff $A \sqsubseteq_\mathcal{T} B$ and $B \sqsubseteq_\mathcal{T} A$. It is not hard to show that $\approx_\mathcal{T}$ is an equivalence relation. For $A \in \mathsf{N}_\mathsf{C}^\mathcal{T}$, let $[A]$ be the equivalence class of A w.r.t. $\approx_\mathcal{T}$, and let $\mathcal{C}_\mathcal{T}$ denote the set of all equivalence classes of $\approx_\mathcal{T}$. Then

- A *choice function* c on $\mathcal{C}_\mathcal{T}$ maps each $[A] \in \mathcal{C}_\mathcal{T}$ to an element $c([A]) \in [A]$;
- The relation "$\preceq_\mathcal{T}$" on $\mathcal{C}_\mathcal{T}$ is defined by setting $[A] \preceq_\mathcal{T} [B]$ if $A \sqsubseteq_\mathcal{T} B$.

Let $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ be a knowledge base and a an individual name. Show that

- "$\preceq_\mathcal{T}$" is well-defined, i.e., if $A' \in [A]$ and $B' \in [B]$, then $A \sqsubseteq_\mathcal{T} B$ if and only if $A' \sqsubseteq_\mathcal{T} B'$.
- A set $R \subseteq \mathsf{N}_\mathsf{C}^\mathcal{T}$ of concept names is a *realization* of a w.r.t. $\mathcal{K}$ if and only if there is a choice function f on $\mathcal{C}_\mathcal{T}$ such that

$$R = \{c([A]) \mid [A] \in \mathcal{C}_\mathcal{T}, a \text{ is an instance of } A \text{ w.r.t. } \mathcal{K}, \text{ and } [A] \text{ is minimal w.r.t. } \preceq_\mathcal{T} \text{ with this property}\}.$$

Exercise 30:

If $\mathcal{D}$ is a concrete domain, we use $\mathcal{ALC}(\mathcal{D})$ to denote the extension of $\mathcal{ALC}$ with the concrete domain $\mathcal{D}$. Show the following:

- If f is an abstract feature, then $\exists f.C$ is equivalent to $\exists f.\top \sqcap \forall f.C$.
- Let $\mathcal{D}$ be a concrete domain with only unary predicates. Let $\mathcal{ALC}(\mathcal{D})^-$ be obtained from $\mathcal{ALC}(\mathcal{D})$ by allowing only concrete features instead of feature chains inside the concrete domain constructor. Prove that for every $\mathcal{ALC}(\mathcal{D})$-concept, there is an equivalent $\mathcal{ALC}(\mathcal{D})^-$-concept.

- Let $\mathcal{N}$ be the concrete domain introduced in the lecture. Define a new concept constructor $\forall u_1, \ldots, u_n.P$, where $u_1, \ldots, u_n$ are feature chains and P is an n-ary predicate, and with the following semantics:

  $$(\forall u_1, \ldots, u_n.P)^{\mathcal{I}} := \{d \in \Delta^{\mathcal{I}} \mid \forall x_1, \ldots, x_n \in \Delta^{\mathcal{N}} : u_i^{\mathcal{I}}(d) = x_i \text{ for all } i \in \{1, \ldots, n\} \text{ implies } (x_1, \ldots, x_n) \in P^{\mathcal{N}}\}.$$

  Prove that the new constructor can be expressed in $\mathcal{ALC}(\mathcal{N})$, i.e., that there is an $\mathcal{ALC}(\mathcal{N})$-concept that is equivalent to $\forall u_1, \ldots, u_n.P$, for all feature chains $u_1, \ldots, u_n$ and all predicates $P \in \Phi^{\mathcal{N}}$.

- Repeat the previous exercise, but this time for the concrete domain $\mathcal{N}'$ that is obtained from $\mathcal{N}$ by dropping the predicates $\geq$, $\leq$, $\geq_n$ and $\leq_n$, for all $n \geq 0$.