7. Exercises for the Course „Description Logics“

Exercise 31:
Prove that, in $\mathcal{ALCO}$, knowledge base consistency can be polynomially reduced to satisfiability of concepts w.r.t. TBoxes.

Exercise 32:
Extend the mappings $\pi_{x}$ and $\pi_{y}$ of $\mathcal{ALC}$-concept descriptions to first-order formulas given in the lecture to $\mathcal{ALCI}$, $\mathcal{ALCQ}$, and $\mathcal{ALCO}$.

Exercise 33:
Reconsider the algorithm for converting an $\mathcal{ALC}$-concept description into negation normal form, which works by exhaustively applying the following rules:

$$
\begin{align*}
\neg T & \rightarrow \bot \\
\neg \bot & \rightarrow T \\
\neg \neg C & \rightarrow C \\
\neg \forall r.C & \rightarrow \exists r.\neg C \\
\neg \exists r.C & \rightarrow \forall r.\neg C \\
\neg (C \cap D) & \rightarrow \neg C \cup \neg D \\
\neg (C \cup D) & \rightarrow \neg C \cap \neg D \\
\neg \forall r.(\forall r.B \cup A) & \subsetneq \forall r.\neg(\exists r.A) \cap \exists r.\exists r.B
\end{align*}
$$

Prove that the algorithm generates an equivalent concept description, and that it needs only polynomial time.

Exercise 34:
Use the tableau algorithm from the lecture to decide whether the following subsumption holds:

$$
\neg \forall r.A \cap \forall r.(\forall r.B \cup A) \subsetneq \forall r.\neg(\exists r.A) \cap \exists r.\exists r.B
$$

Exercise 35:
Extend the proof of Lemma 3.4 (local correctness) to the $\cap$-rule and the $\forall$-rule.

Exercise 36:
Reconsider the proof of Point 1 of Lemma 3.5 (soundness). Fill in the remaining cases in the inductive proof that $I_A$ satisfies every assertion $C(x) \in A$. 