8. Exercises for the Course  
„Description Logics“

Exercise 37:
Extend the tableau algorithm given in the lecture such that it can decide ABox consistency in the description logic ALCN, i.e., ALC extended with unqualified number restrictions (≥ n r) and (≤ n r). In particular,
- find new tableau rules if necessary;
- extend the proof of local correctness;
- adapt/extend the proofs of soundness and completeness;

Exercise 38:
Let (P_1, >_1) and (P_2, >_2) be partial orders, P = P_1 × P_2, and let (P, >_{lex}) be defined as in the lecture. Prove that if (P_1, >_1) and (P_2, >_2) are well-founded, then so is (P, >_{lex}).

Exercise 39:
You are experiencing a busy semester and there are awfully many tasks to be accomplished. You have listed all of them in your organizer, along with a priority, which is a number between 1 (lowest) and 100 (highest). You address these task in order of highest priority, never working on more than one task at a time. While working on a task, it may happen that you have to address a number of subtasks. If this is the case, you delete the current task from your organizer, and add the subtasks, all of them with strictly higher priority than the task just deleted.
Use a multiset order to prove that you will eventually accomplish all tasks.

Exercise 40:
Prove Lemma 3.8 from the lecture.

Exercise 41:
Complete the proof of Lemma 3.10 by treating the ⊔-rule and the ⊓-rule.