9. Exercises for the Course „Description Logics“

Exercise 42:
Give an inductive definition of
(a) the size $|C|$ of a concept description $C$ and
(b) the set $\text{sub}(C)$ of subdescriptions of $C$.

Prove that for all concept descriptions $C$, the cardinality of $\text{sub}(C)$ is bounded by $|C|$.

Exercise 43:
Show that the size $|C|_T$ of a concept $C$ w.r.t. to an acyclic TBox $T$, as defined in the lecture, is well-defined.

Exercise 44:
Reconsider the proof of Point 1 of Lemma 3.15 (soundness). Fill in the remaining case $C = \neg A$ in the inductive proof that $I_A$ satisfies every assertion $C(x) \in A$.

Exercise 45:
(Partially) prove the analogue of Lemma 3.10 for the case with acyclic TBoxes, i.e., if the set of ABoxes $\mathcal{M}'$ is obtained from $\mathcal{M}$ by an application of a tableau rule, then $\mathcal{M}' \prec \mathcal{M}$. Treat the $\forall$-rule and the $\equiv_1$-rule.

Exercise 46:
Complete the proof of Lemma 4.2 by showing that for all $\mathcal{ALC}$-concept descriptions $C$ and all $(d_0, r_1, \ldots, r_m, d_m) \in \Delta^I$, we have $(d_0, r_1, \ldots, r_m, d_m) \in \Delta^\mathcal{I}$ iff $d_m \in C^\mathcal{I}$. 