Lehrstuhl für Automatentheorie
Institut für Theoretische Informatik, TU Dresden
Prof. Dr. F. Baader

Nöthnitzer Str. 46
01187 Dresden
Tel.: 0351/463–38237
Tel.: 0351/463–39171

10. Exercises for the Course „Description Logics“

Exercise 47:

Let $\mathcal{K} = (\mathcal{A}_0, \mathcal{T})$ be an $\mathcal{ALC}$-knowledge base, with $\mathcal{T}$ a general TBox. The *precompletion* of $\mathcal{K}$ is the set of ABoxes $\mathcal{M}$ that is produced by the tableau algorithm when starting with the set of ABoxes $\{\mathcal{A}_0\}$ and exhaustingly applying all rules except the $\exists$-rule. Do the following:

(a) Show that $\mathcal{K}$ is consistent iff there is an open $\mathcal{A} \in \mathcal{M}$ such that for all individual names a occurring in $\mathcal{A}$, the concept $C^a_{\mathcal{A}} := \bigsqcap_{C(a) \in \mathcal{A}} C$ is satisfiable w.r.t. $\mathcal{T}$.

Hint: For the “if” direction, proceed as follows. The correctness of the tableau algorithm for $\mathcal{ALC}$ implies that, if $C^a_{\mathcal{A}}$ is satisfiable, then exhaustively applying (all!) rules to the set of ABoxes $\{\{C^a_{\mathcal{A}}(a)\}\}$ yields a set $\mathcal{M}'$ that contains an open and complete ABox. Show how to join all these ABoxes to obtain an open and complete tableau for $\mathcal{A}$ and conclude that $\mathcal{A}_0$ is consistent w.r.t. $\mathcal{T}$.

(b) Use the result from (a) to prove that ABox consistency in $\mathcal{ALC}$ can be decided in deterministic exponential time (ExpTime).

Exercise 48:

For each of the following languages of binary trees over the alphabet $\Sigma = \{a, b\}$, define a looping tree automaton that accepts the language:

(a) The set of all trees that contain a branch (starting at the root) in which all nodes are labelled with a;

(b) The set of all trees T that do not contain nodes n_0, n_1, n_2 such that (i) $n_1 = n_0 i$ for some $i \in \{0, 1\}$, (ii) $n_2 = n_i j$ for some $j \in \{0, 1\}$, and (iii) $T(n_0) = T(n_1) = T(n_2) = a$.

Exercise 49:

Recall that a *propositional Horn clause* is a formula of the form $p_1, \ldots, p_k \rightarrow p$, where $p_1, \ldots, p_k$ are propositional letters and p is a propositional letter or $\bot$. A *Horn formula* is a set of Horn clauses. Also recall that the satisfiability of Horn formulas can be decided in linear time.

Show that the emptiness problem for looping tree automata can be decided in linear time by giving a linear-time reduction to the satisfiability of Horn formulas.

You may assume that the automaton is given in a form such that, for any state q, we can retrieve all the transitions with q as left-most component in linear time.