Theorem Proving and Equality

Exercise 1 *Change into the prenex normal form and skolemize:*

1. $\forall x(Person(x) \to \exists y Parent(y, x))$
2. $\forall x_0(\forall x_1 R_1(x_1, x_0) \to \exists x_1 R_2(x_0, x_1))$

Exercise 2 *Put into skolemized clauses:*

1. $\exists x.(Dog(x) \land Owns(Jack, x))$
2. $\forall x.(AnimalLover(x) \to (\exists y.Animal(y) \land Owns(x, y)))$
3. $\forall x.(\exists y.Dog(y) \land Owns(x, y) \to AnimalLover(x))$
4. $\forall x.(AnimalLover(x) \to (\forall y.Aminal(y) \to \neg Kills(x, y)))$

Exercise 3 *Prove with the help of $\mathcal{B}$ that the following formula is a FOL-theorem:*
$\forall x[(P(x) \to Q(x)) \land (Q(x) \to R(x)) \land (P(x) \to \neg R(x))] \to \neg P(c)$

Exercise 4 *Prove the following claim:*
If S is a finite set of clauses and S' is a saturation of S by the inferences from a finite set of inference rules Γ, and D is a clause in S', then D' has a finite derivation from S (i.e. there is a finite sequence of clauses $D_1, \ldots, D_n, D$, such that each clause in this sequance is either in S or is obtained form other clauses by one of the rules in Γ).

Exercise 5 *Use inferences in $\mathcal{B}$ (binary resolution and positive factoring) to prove that $a \approx c$ follows from $b \approx a$ and $b \approx c$.*

Exercise 6 *Formalize and prove using resolution:*

Mary's boyfriend is a student and he owns a car.
John is the only student who owns a car.
Therefore John is Mary's boyfriend.