Theorem Proving and Equality

Exercise 1 *Prove with the help of $\mathcal{B}$ that the following formula is a FOL-theorem:*

$\forall x[(P(x) \rightarrow Q(x)) \wedge (Q(x) \rightarrow R(x)) \wedge (P(x) \rightarrow \neg R(x))] \rightarrow \neg P(c)$

Exercise 2 *Use inferences in $\mathcal{B}$ (binary resolution and positive factoring) to prove that $a \approx c$ follows from $b \approx a$ and $b \approx c$.*

Exercise 3 *Formalize and prove using resolution. You can also use paramodulation inference:*

Mary's boyfriend is a student and he owns a car.
John is the only student who owns a car.
Therefore John is Mary's boyfriend.

Exercise 4 *Formalize and prove using resolution and paramodulation:*

Prof. Smith has only very good students at his lectures.
John and Mary attend the same lectures.
Mary attends Prof. Smith's lecture.
Therefore John is a very good student.

Exercise 5 *Formalize and prove using resolution and paramodulation:*

Every part is made either by manufacturer A or by manufacturer B.
All fragile parts are stored in the warehouse of their manufacturer.
Manufacturer B cannot manufacture titanium parts.
The part I need is fragile and made of titanium.
Therefore, the part I need is in the A's warehause.