Theorem Proving and Equality

Exercise 7 *Assume maximal selection function and try to saturate with ordered resolution system $\mathcal{O}$ the following set of clauses:*

$\{\neg S(x_1) \vee \neg P(x_1, b),\ \neg P(a, x_2) \vee Q(x_2, x_2),\ \neg Q(x_3, y) \vee R(x_3) \vee S(x_3)\}$.

What can you conclude about the consistency of this set and about similar sets of clauses ?

Exercise 8 *If a set of clauses is such that each of the clauses contains at least one maximal positive literal, is it consistent or it may still be inconsistent?*

Exercise 9 *Use ordered resolution in deciding if the following set is consistent:*

$\{A_1, \neg A_1 \vee A_2, \neg A_2 \vee A_3,$
$B_1, \neg B_1 \vee B_2, \neg B_2 \vee B_3,$
$C_1, \neg C_1 \vee C_2, \neg C_2 \vee C_3,$
$\neg A_3 \vee \neg B_3 \vee \neg C_3, \}$

What inferences are preformed depending on selection function?

Exercise 10 *A Horn clause is a clause with at most one positive literal. Prove the refutation soundness and completeness of the following rule for Horn clauses:*

Positive Unit Resolution:

$$\frac{A \quad D \vee \neg B}{D\sigma}$$

where $\sigma = mgu(A, B)$.

Exercise 11 *Unit Resolution is a binary resolution where one of the premises must be a unit clause (i.e. a clause containing one literal). Using Unit Resolution prove that* $\exists x_5 . \neg S(x_5) \wedge R(x_5)$ *follows from the following set of universally quantified clauses:*

$\{\neg S(x_1) \vee \neg P(x_1, b),\ \neg P(a, x_2) \vee Q(x_2, x_2),\ \neg Q(x_3, y) \vee R(x_3) \vee S(x_3),\ P(a, x_4)\}$.

Exercise 12 *Solve the above problem with Ordered Resolution. (In Otter hyperresolution. Try it with sos and notice that sos is not complete, because it depends on a chosen order of literals.)*