Lehrstuhl für Automatentheorie
Institut für Theoretische Informatik, TU Dresden
Prof. Dr. F. Baader

Nöthnitzer Str. 46
01187 Dresden
Tel.: 0351/463–38237
Tel.: 0351/463–39171

4. Exercises for the Course „Complexity and Logic“

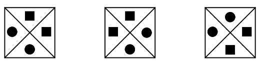
Exercise 15:

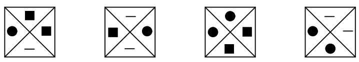
Give a proof of Corollary 2.3 from the lecture: show that every language that is decidable by an NTM is also decidable by a DTM.

Hint: Use the Turing machine from Exercise 15, find an appropriate adaptation of the construction from the proof of Theorem 2.2, and use König's lemma.

Exercise 16:

Check whether or not the following domino systems have a solution:

(a) 

(b) 

Exercise 17:

The domino systems for the lecture require the tiling of the first quadrant of the plane: solutions τ have domain $\mathbb{N} \times \mathbb{N}$. By changing the domain of τ, we obtain versions of the domino problem that require tilings of the full plane $\mathbb{Z} \times \mathbb{Z}$ or of an n-square $\{0, \ldots, n-1\} \times \{0, \ldots, n-1\}$. Prove the following:

- if a domino system $\mathcal{D} = (D, H, V)$ can tile the n-square, for each $n \geq 0$, then $\mathcal{D}$ can tile the first quadrant of the plane. Hint: Use König's lemma.
- if a domino system $\mathcal{D} = (D, H, V)$ can tile the first quadrant of the plane, then it can tile the whole plane.

Exercise 18:

Let $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{\mathsf{acc}}, q_{\mathsf{rej}})$ be a TM defined as follows:

- $Q = \{q_0, q_1, q_2, q_{\mathsf{acc}}, q_{\mathsf{rej}}, q_{\mathsf{loop}}\}$;
- $\Sigma = \Gamma = \{0, 1, \#, \sqcup\}$;
- $\delta(q_0, 1) = (q_0, 0, R), \quad \delta(q_0, 0) = (q_1, 0, R), \quad \delta(q_0, \#) = (q_0, \#, R)$;
- $\delta(q_1, 1) = (q_1, 1, R), \quad \delta(q_1, 0) = (q_1, 0, R), \quad \delta(q_1, \#) = (q_{\mathsf{loop}}, \#, L)$, $\delta(q_1, \sqcup) = (q_2, \#, L)$;
- $\delta(q_2, 1) = (q_2, 1, L), \quad \delta(q_2, 0) = (q_2, 0, L), \quad \delta(q_2, \#) = (q_0, \#, R)$;
- $\delta(q_{\mathsf{loop}}, \sqcup) = \delta(q_{\mathsf{loop}}, \sqcup, R)$.

For all remaining combinations $q \in Q$ and $a \in \Sigma$, $\delta(q, a) = q_{\mathsf{rej}}$. Construct the domino system $\mathcal{D}$ corresponding to M and the input #1101. Construct a solution to $\mathcal{D}$.