

## 6. Exercises for the Course „Complexity and Logic“

### Exercise 24:

Prove the space compression theorem (Theorem 3.8 from the lecture): for all  $\varepsilon \in \mathbb{R}^+$  and all  $S : \mathbb{N} \rightarrow \mathbb{R}^+$ , we have  $D\text{Space}(S) \subseteq D\text{Space}(\max(n, \varepsilon \cdot S(n)))$ .

### Exercise 25:

Complete the proof of Lemma 3.10 from the lecture: let  $S$ ,  $M$ , and  $X$  be as in the proof sketch given in the lecture. Show that, for words  $w \in \Sigma^* \setminus X$ , the question “ $w \in L(M)$ ?” can be decided by simulating  $M$  using only space  $S(|w|)$ .

### Exercise 26:

Prove Theorem 3.12 from the lecture, i.e., the gap theorem for time: for every total computable function  $g$  with  $g(n) \geq n$ , there is a total computable function  $T$  with  $D\text{Time}(T) = D\text{Time}(g \circ T)$ .