6. Exercises for the Course „Complexity and Logic“

Exercise 24:
Prove the space compression theorem (Theorem 3.8 from the lecture): for all \( \varepsilon \in \mathbb{R}^+ \) and all \( S : \mathbb{N} \to \mathbb{R}^+ \), we have \( \text{DSpace}(S) \subseteq \text{DSpace}(\max(n, \varepsilon \cdot S(n))) \).

Exercise 25:
Complete the proof of Lemma 3.10 from the lecture: let \( S, M, \) and \( X \) be as in the proof sketch given in the lecture. Show that, for words \( w \in \Sigma^* \setminus X \), the question “\( w \in L(M)? \)” can be decided by simulating \( M \) using only space \( S(|w|) \).

Exercise 26:
Prove Theorem 3.12 from the lecture, i.e., the gap theorem for time: for every total computable function \( g \) with \( g(n) \geq n \), there is a total computable function \( T \) with \( \text{DTIME}(T) = \text{DTIME}(g \circ T) \).