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7. Exercises for the Course „Complexity and Logic“

Exercise 27:

Prove the following:

(a) The functions $2n$ and n^2 are time constructible.

(b) The functions 2^n and $n!$ are space constructible.

(c) If the functions f and g are space constructible, then so are $f + g$, $f \cdot g$, and 2^f.

Exercise 28:

Complete the proof of Theorem 3.24 by showing how a Turing machine can construct the computation graph $G[M, T(n)]$ in time $2^{\mathcal{O}(T(n))}$ if T is space constructible and satisfies $T(n) \geq n$ for all $n \in \mathbb{N}$.

Exercise 29:

In the lecture, it was explained that non-deterministic transitions of a Turing machines can be thought of as "guessing". For example, a word $u \in \Sigma^m$ can be guessed by m consecutive transitions, each one non-deterministically producing a symbol from Σ on the tape.

Consider NTMs N that are $O(n)$-time bounded. Can such NTMs perform the following when started on an input of length n?

(a) guess a natural number between 0 and n;

(b) guess a natural number between 0 and 2^n;

(c) guess a word from $\{a, b\}^*$ of length 2^n;

(d) guess a rational number between 0 and n;

(e) guess a word from $\mathbb{N}^*$ of length n.

Exercise 30:

Prove the following. It is allowed to use theorems from the lecture.

(a) $\mathsf{DTime}(2^n) = \mathsf{DTime}(2^{n+1})$;

(b) $\mathsf{DTime}(2^n) \subsetneq \mathsf{DTime}(2^{3n})$;

(c) $\mathsf{NTime}(n) \subsetneq \mathsf{PSpace}$.