Exercise 31:
Show that the following problem is in PSPACE: given a non-deterministic finite automaton (NFA) $A$ over an alphabet $\Sigma$, is $L(A) = \Sigma^*$?

Hint: prove that, if $L(A) \neq \Sigma^*$ and $A$ has $n$ states, then there exists a word $w \in \Sigma^*$ of length at most $2^n$ such that $w \notin L(A)$. Then use this fact to give a non-deterministic algorithm whose space consumption is polynomially bounded. Finally, apply Savitch’s theorem.

Exercise 32:
Prove that the following problems can be solved in polynomial time:
- given a Boolean formula $\varphi$ using variables $p_1, \ldots, p_k$ and a truth assignment $w \in \{0, 1\}^k$, check whether $w$ is a model of $\varphi$.
- given an $\mathcal{ALCF}$ concept $C$ (as introduced in the lecture) and a finite interpretation $I$, check whether $C^I \neq \emptyset$.

Exercise 33:
An acyclic finite automaton (AFA) is a non-deterministic finite automaton whose transition relation is acyclic, i.e., there is no sequence $a_0, \ldots, a_{n-1}$ of symbols and $q_0, \ldots, q_n$ of states such that $q_0 = q_n$ and $q_{i+1} \in \delta(q_i, a_i)$ for all $i < n$.

(In other words, AFAs are exactly the non-deterministic finite automata that accept finite languages). Prove that the following problem is in NP: given AFAs $A_1$ and $A_2$, does $L(A_1) \not\subseteq L(A_2)$ hold?

Exercise 34:
Prove Theorem 4.6 from the lecture:
(a) $B \in P$ and $A \leq_p B$ implies $A \in P$;
(b) $B \in NP$ and $A \leq_p B$ implies $A \in NP$;
(c) $A \leq_p B$ and $B \leq_p C$ implies $A \leq_p C$. 