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8. Exercises for the Course „Complexity and Logic“

Exercise 31:

Show that the following problem is in PSPACE: given a non-deterministic finite automaton (NFA) $\mathcal{A}$ over an alphabet Σ, is $L(\mathcal{A}) = \Sigma^*$?

Hint: prove that, if $L(\mathcal{A}) \neq \Sigma^*$ and $\mathcal{A}$ has n states, then there exists a word $w \in \Sigma^*$ of length at most 2^n such that $w \notin L(\mathcal{A})$. Then use this fact to give a non-deterministic algorithm whose space consumption is polynomially bounded. Finally, apply Savitch's theorem.

Exercise 32:

Prove that the following problems can be solved in polynomial time:

- given a Boolean formula φ using variables $p_1, \ldots, p_k$ and a truth assignment $w \in \{0,1\}^k$, check whether w is a model of φ.
- given an $\mathcal{ALCF}$ concept C (as introduced in the lecture) and a finite interpretation $\mathcal{I}$, check whether $C^{\mathcal{I}} \neq \emptyset$.

Exercise 33:

An *acyclic* finite automaton (AFA) is a non-deterministic finite automaton whose transition relation is acyclic, i.e., there is no sequence $a_0, \ldots, a_{n-1}$ of symbols and $q_0, \ldots, q_n$ of states such that $q_0 = q_n$ and

$$q_{i+1} \in \delta(q_i, a_i) \text{ for all } i < n.$$

(In other words, AFAs are exactly the non-deterministic finite automata that accept finite languages). Prove that the following problem is in NP: given AFAs $\mathcal{A}_1$ and $\mathcal{A}_2$, does $L(\mathcal{A}_1) \not\subseteq L(\mathcal{A}_2)$ hold?

Exercise 34:

Prove Theorem 4.6 from the lecture:

(a) $B \in \mathrm{P}$ and $A \leq_p B$ implies $A \in \mathrm{P}$;

(b) $B \in \mathrm{NP}$ and $A \leq_p B$ implies $A \in \mathrm{NP}$;

(c) $A \leq_p B$ and $B \leq_p C$ implies $A \leq_p C$.