

## 9. Exercises for the Course „Complexity and Logic“

### Exercise 35:

An *n-time bounded NTM* is a non-deterministic Turing machine that, on input  $w$ , makes at most  $|w|$  steps. Prove that the following problem is NP-complete: given an *n-time bounded NTM*  $M$  and an input  $w$ , decide whether  $M$  accepts  $w$ .

### Exercise 36:

Complete the 3rd step in the proof of Theorem 4.11 by proving that, if a formula  $\psi_1$  is transformed into a formula  $\psi_2$  in this step, then  $\psi_1$  is satisfiable iff  $\psi_2$  is satisfiable. Also show that these two formulas are not equivalent.

### Exercise 37:

Let  $\varphi$  be a 3-formula. An  $\neq$ -assignment to the variables in  $\varphi$  is one where each clause contains two literals with unequal truth values. In other words, a  $\neq$ -assignment satisfies  $\varphi$  without assigning three true literals in any clause. Let  $\neq 3\text{SAT}$  be the set of 3-formulas that have an  $\neq$ -assignment. Show that  $\neq 3\text{SAT}$  is NP-complete.

Hint: use a reduction from 3SAT that translates every 3SAT clause into two  $\neq 3\text{SAT}$  clauses. Showing the correctness of the translation is likely to involve showing the following: the negation of a  $\neq$ -assignment satisfying a formula  $\varphi$  is also a  $\neq$ -assignment that satisfies  $\varphi$ .

### Exercise 38:

Prove that 2-colorability is in P.

### Exercise 39:

Complete the proof of Theorem 4.17 from the lecture: show that a mapping  $h : V_1 \rightarrow V_2$  is a homomorphism from  $G_1$  to  $G_2$  if and only if  $h$  is a homomorphism from  $\widehat{G}_1$  to  $\widehat{G}_2$ .

### Exercise 40:

Complete the proof of Theorem 4.19 from the lecture: show that the containment of conjunctive queries can be polynomially reduced to conjunctive query answering.