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10. Exercises for the Course „Complexity and Logic“

Exercise 41:

Complete the proof of Theorem 4.20 from the lecture: show that there exists a homomorphism $h : G_1 \rightarrow G_2$ with $h(u_0) = v_0$ iff x_1 is an answer for K w.r.t. $\mathcal{I}$.

Exercise 42:

Show that, if P = NP, then there exists a polynomial time algorithm that, given a formula φ of propositional logic, produces a satisfying assignment for φ (if φ is satisfiable).

Exercise 43:

Which of the following QBFs is valid?

(a) $\forall p_1 \exists p_2 \forall p_3 (p_2 \rightarrow (p_1 \vee p_3))$

(b) $\forall p_1 \exists p_2 \forall p_3 ((p_1 \vee p_2) \rightarrow p_3)$

(c) $\forall p_1 \forall p_2 \forall p_3 \exists q_1 \exists q_2 (q_1 \leftrightarrow ((p_1 \vee p_2) \wedge (\neg p_1 \vee \neg p_2)) \wedge (q_2 \leftrightarrow ((q_1 \vee p_3) \wedge (\neg q_1 \vee \neg p_3)))$

Exercise 44:

NICHT WEGLASSEN, SONST NAECHSTE AUFGABE BLOED. Let $\psi = Q_1 p_1 \ldots Q_n p_n . \varphi$ be a QBF. A *witness tree* for ψ is a labelled tree $T = (V, E, \ell)$ such that

- $\ell : V \rightarrow \{0, 1\}$ is a node labelling function;
- if $v \in V$ is on level $i < n$ of T (the root is on level 0), then v has 1 successor if $Q_{i+1} = \exists$ and 2 successors if $Q_{i+1} = \forall$;
- if $v \in V$ is on level n of T and the path from the root to v is $v_0, \ldots, v_n$, then v has no successors and the following mapping is a truth assignment that makes φ true:

$$\tau(p_i) := \ell(v_i) \text{ for } 1 \leq i \leq n$$

Prove that a QBF ψ is valid iff there exists a witness tree T for ψ.

Exercise 45:

Prove Lemma 5.11 from the lecture:

- The QBF ψ is valid iff C_ψ is satisfiable;
- C_ψ can be constructed in polynomial time from ψ.

Exercise 46:

Formulas of $FO^=$, the first-order logic of equality, are inductively defined as follows:

- If u and v are variables, then $(u = v)$ is a formula;
- if φ and ψ are formulas and u is a variable, then the following are also formulas $\neg\varphi$, $\varphi \wedge \psi$, $\varphi \vee \psi$, $\exists u.\varphi$, and $\forall u.\varphi$.

A $FO^=$ formula is a *sentence* if it does not have free variables. The semantics is defined as usual in first-order logic with equality. Note that models consist only of a universe (of arbitrary, but non-zero cardinality) since there are no function symbols and no uninterpreted predicate symbols in the syntax.

The first-order theory of equality is the set of all $FO^=$ sentences that are valid. Show by a QBF reduction that the first-order theory of equality is PSPACE-hard.