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4. Exercises for the Course 'Description Logics'

Exercise 12:

Let $\mathcal{I}$, $\mathcal{J}$ be interpretations such that $\Delta^{\mathcal{I}} \cap \Delta^{\mathcal{J}} = \emptyset$. The *disjoint union* $\mathcal{I} \oplus \mathcal{J}$ of $\mathcal{I}$ and $\mathcal{J}$ is the interpretation defined as follows:

- $\Delta^{\mathcal{I} \oplus \mathcal{J}} := \Delta^{\mathcal{I}} \uplus \Delta^{\mathcal{J}}$;
- $A^{\mathcal{I} \oplus \mathcal{J}} := A^{\mathcal{I}} \uplus A^{\mathcal{J}}$ for all concept names A;
- $r^{\mathcal{I} \oplus \mathcal{J}} := r^{\mathcal{I}} \uplus r^{\mathcal{J}}$ for all role names r.

Let $\mathcal{T}$ be a general TBox. Show that the set of models of $\mathcal{T}$ is *closed under disjoint unions*, i.e., if $\mathcal{I}$ and $\mathcal{J}$ are models of $\mathcal{T}$ with $\Delta^{\mathcal{I}} \cap \Delta^{\mathcal{J}} = \emptyset$, then $\mathcal{I} \oplus \mathcal{J}$ is a model of $\mathcal{T}$.
Hint: for the next two exercises, you might want to use disjoint unions.

Exercise 13:

Prove or disprove the following (for the description logic $\mathcal{ALC}$):

(a) There is a TBox that has no models at all.

(b) There is an ABox that has only finite models.

(c) Every TBox has either no models at all or infinitely many models.

(d) There is an ABox $\mathcal{A}$ such that all models are either infinite or contain a cycle (when viewed as a graph).

(e) For every TBox $\mathcal{T}$, there is an equivalent TBox $\mathcal{T}'$ that contains only a single GCI (where two TBoxes are equivalent if they have the same models).

Exercise 14:

Revisit the procedure for expanding TBoxes given in the proof of Proposition 2.9. Prove that

(a) this procedure always terminates, and

(b) that it returns a TBox that is equivalent to its input.

Hint for proving termination: count, for each concept name A, the number of concept names (directly or indirectly) used in the definition of A.

Exercise 15:

Which of the following statements are true? Give reasons.

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\begin{array}{llcllllcl}
(a) & \forall r.(A \sqcap B) & \sqsubseteq & \forall r.A \sqcap \forall r.B & \qquad & (e) & \exists r.(A \sqcap B) & \sqsubseteq & \exists r.A \sqcap \exists r.B \\
(b) & \forall r.A \sqcap \forall r.B & \sqsubseteq & \forall r.(A \sqcap B) & & (f) & \exists r.A \sqcap \exists r.B & \sqsubseteq & \exists r.(A \sqcap B) \\
(c) & \forall r.(A \sqcup B) & \sqsubseteq & \forall r.A \sqcup \forall r.B & & (h) & \exists r.(A \sqcup B) & \sqsubseteq & \exists r.A \sqcup \exists r.B \\
(d) & \forall r.A \sqcup \forall r.B & \sqsubseteq & \forall r.(A \sqcup B) & & (g) & \exists r.A \sqcup \exists r.B & \sqsubseteq & \exists r.(A \sqcup B)
\end{array}
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Exercise 16:

Let $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ be a consistent knowledge base. We write $C \sqsubseteq_{\mathcal{K}} D$ if $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ for all models $\mathcal{I}$ of $\mathcal{K}$. Prove that for all $\mathcal{ALC}$-concepts C and D, we have $C \sqsubseteq_{\mathcal{K}} D$ iff $C \sqsubseteq_{\mathcal{T}} D$.

Hint: Use disjoint unions.