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7. Exercises for the Course 'Description Logics'

Exercise 26:

Reconsider the proof of Point 1 of Lemma 3.14 (local correctness). Prove the remaining case for the $\equiv_2$-rule .

Exercise 27:

Reconsider the proof of of Lemma 3.15 (soundness and completeness). Fill in the remaining case $C = \neg A$ in the inductive proof that $\mathcal{I}_\mathcal{A}$ satisfies every assertion $C(x) \in \mathcal{A}$.

Exercise 28:

In this exercise we compare the initial tableaux algorithm (with eager expansion) and the one with the $\equiv_1$- and $\equiv_2$-rules (lazy expansion). If we apply both methods to test whether C is satisfiable w.r.t. the following TBox:

$$
\begin{array}{rrcllcl}
\mathcal{T} = \{ & C & \equiv & A_1 \sqcap B_1 \sqcap D \sqcap \forall r.E, & & & \\
& D & \equiv & \neg A_1 \sqcap \neg B_1 \sqcap \exists r.\neg E, & E & \equiv & A_1 \sqcup B_1, \\
& A_1 & \equiv & A_2 \sqcup A_3, & B_1 & \equiv & B_2 \sqcup B_3, \\
& A_2 & \equiv & \exists r.B_2, & B_2 & \equiv & \exists r.A_3, \\
& A_3 & \equiv & \exists r.Z, & B_3 & \equiv & \exists r.Y \}.
\end{array}
$$

a) What is the maximal number of complete ABoxes obtained in the set of ABoxes by eager expansion? What is the minimal number for lazy expansion?

b) What is the maximal number of rule applications by eager expansion? What is the minimal number for lazy expansion?

c) What is the maximal number of assertions in the biggest complete ABox obtained by eager expansion? What is the minimal number for lazy expansion?

Exercise 29:

Consider the definition of *size of a concept description w.r.t. an acyclic TBox* from the lecture. Given a TBox $\mathcal{T}$:

- $|\bot|_\mathcal{T} := |\top|_\mathcal{T} := |P|_\mathcal{T} := |\neg P|_\mathcal{T} := 1$ for primitive concepts P.
- $|C \sqcap D|_\mathcal{T} := 1 + |C|_\mathcal{T} + |D|_\mathcal{T} =: |C \sqcup D|_\mathcal{T}$
- $|\forall r.C|_\mathcal{T} := 1 + |C|_\mathcal{T} =: |\exists r.C|_\mathcal{T}$
- if A is a defined concept, then
  - $|A|_\mathcal{T} := 1 + |C|_\mathcal{T}$, where $A \equiv C \in \mathcal{T}$
  - $|\neg A|_\mathcal{T} := 1 + |\sim C|_\mathcal{T}$, where $A \equiv C \in \mathcal{T}$

Prove that this defines a unique natural number $|C|_\mathcal{T}$ to every concept description in NNF. Use the fact that $\mathcal{T}$ is acyclic.