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8. Exercises for the Course 'Description Logics'

Exercise 30:

Complete the proof of Lemma 4.2 by showing that for all $\mathcal{ALC}$-concept descriptions C and all tuples $(d_0, r_1, \ldots, r_m, d_m) \in \Delta^{\hat{\mathcal{I}}}$, it holds that $(d_0, r_1, \ldots, r_m, d_m) \in C^{\hat{\mathcal{I}}}$ iff $d_m \in C^{\mathcal{I}}$.

Exercise 31:

For the following looping tree automata on binary Σ-trees, with $\Sigma = \{a, b\}$, describe the language that they accept:

(a) $(\{q_1, q_2\}, \Sigma, \{q_1\}, \Delta)$, where Δ is: $(q_1, a) \to (q_1, q_2), (q_2, b) \to (q_2, q_2)$.

(b) $(\{q_1, q_2\}, \Sigma, \{q_1, q_2\}, \Delta)$, where Δ is: $(q_1, a) \to (q_1, q_2), (q_2, b) \to (q_2, q_1)$.

Exercise 32:

For each of the following languages of binary trees over the alphabet $\Sigma = \{a, b\}$, define a looping tree automaton that accepts the language:

(a) The set of all trees that contain a branch (starting at the root) in which all nodes are labelled with a;

(b) The set of all trees T that do not contain nodes n_0, n_1, n_2 such that (i) $n_1 = n_0 i$ for some $i \in \{0, 1\}$, (ii) $n_2 = n_1 j$ for some $j \in \{0, 1\}$, and (iii) $T(n_0) = T(n_1) = T(n_2) = a$.

Exercise 33:

Show that there is no looping tree automaton on binary $\{a, b\}$-trees that accepts the set of all trees that contain a branch with infinitely many nodes labeled with a.

Exercise 34:

Recall that a *propositional Horn clause* is a formula of the form $p_1, \ldots, p_k \to p$, where $p_1, \ldots, p_k$ are propositional letters and p is a propositional letter or $\bot$. A *Horn formula* is a set of Horn clauses. Also recall that the satisfiability of Horn formulas can be decided in linear time.

Show that the emptiness problem for looping tree automata can be decided in linear time by giving a linear-time reduction to the satisfiability of Horn formulas.