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9. Exercises for the Course 'Description Logics'

Exercise 35:

Let $\mathcal{K} = (\mathcal{A}_0, \mathcal{T})$ be an $\mathcal{ALC}$-knowledge base, with $\mathcal{T}$ a general TBox. The *precompletion* of $\mathcal{K}$ is the set of ABoxes $\mathcal{M}$ that is produced by the tableau algorithm when starting with the set of ABoxes $\{\mathcal{A}_0\}$ and exhaustingly applying all rules except the $\exists$-rule. Do the following:

(a) Show that $\mathcal{K}$ is consistent iff there is an open $\mathcal{A} \in \mathcal{M}$ such that for all individual names a occurring in $\mathcal{A}$, the concept $C^a_{\mathcal{A}} := \bigsqcap_{C(a)\in\mathcal{A}} C$ is satisfiable w.r.t. $\mathcal{T}$.

Hint: For the "if" direction, proceed as follows. The correctness of the tableau algorithm for $\mathcal{ALC}$ implies that, if $C^a_{\mathcal{A}}$ is satisfiable, then exhaustively applying (all!) rules to the set of ABoxes $\{\{C^a_{\mathcal{A}}(a)\}\}$ yields a set $\mathcal{M}'$ that contains an open and complete ABox. Show how to join all these ABoxes to obtain an open and complete tableau for $\mathcal{A}$ and conclude that $\mathcal{A}_0$ is consistent w.r.t. $\mathcal{T}$.

(b) Use the result from (a) to prove that ABox consistency in $\mathcal{ALC}$ can be decided in deterministic exponential time (ExpTime).

Exercise 36:

Reconsider the claim: for all $D \in S_{C,\mathcal{T}}$ we have $D \in R(u) \Rightarrow u \in D^{\mathcal{I}_R}$. Show the claim by induction on the structure of D for the missing cases:

- $D = D_1 \sqcap D_2$ and
- $D = \exists r.E$.

Exercise 37:

Show that the transformation of $\mathcal{FL}_0$-concept descriptions into normal form requires only polynomial time.

Exercise 38:

Show that subsumption in $\mathcal{FL}_0$ w.r.t. acyclic TBoxes is in co-NP by giving a polytime reduction from this problem to the inclusion problem for acyclic finite automata (which is in co-NP).