Lehrstuhl für Automatentheorie
Institut für Theoretische Informatik, TU Dresden
Prof. Dr. F. Baader

Nöthnitzer Str. 46
01187 Dresden
Tel.: 0351/463–39167

12. Exercises for the Course 'Description Logics'

Exercise 42:

If $\mathcal{D}$ is a concrete domain, we use $\mathcal{ALC}(\mathcal{D})$ to denote the extension of $\mathcal{ALC}$ with the concrete domain $\mathcal{D}$. Show the following:

(a) If f is an abstract feature, then $\exists f.C$ is equivalent to $\exists f.\top \sqcap \forall f.C$.

(b) Let $\mathcal{D}$ be a concrete domain with only unary predicates. Let $\mathcal{ALC}(\mathcal{D})^-$ be obtained from $\mathcal{ALC}(\mathcal{D})$ by allowing only concrete features instead of feature chains inside the concrete domain restictions. Prove that for every $\mathcal{ALC}(\mathcal{D})$-concept, there is an equivalent $\mathcal{ALC}(\mathcal{D})^-$-concept.

(c) Let $\mathcal{N}$ be the concrete domain introduced in Section 6.1 in the lecture. Define a new concept constructor $\forall u_1, \ldots, u_n.P$, where $u_1, \ldots, u_n$ are feature chains and P is an n-ary predicate, and with the following semantics:

$$(\forall u_1, \ldots, u_n.P)^{\mathcal{I}} := \{d \in \Delta^{\mathcal{I}} \mid \forall x_1, \ldots, x_n \in \Delta^{\mathcal{N}} : u_i^{\mathcal{I}}(d) = x_i \text{ for all } i \in \{1, \ldots, n\} \text{ implies } (x_1, \ldots, x_n) \in P^{\mathcal{N}}\}.$$

Prove that the new constructor can be expressed in $\mathcal{ALC}(\mathcal{N})$, i.e., that there is an $\mathcal{ALC}(\mathcal{N})$-concept that is equivalent to $\forall u_1, \ldots, u_n.P$, for all feature chains $u_1, \ldots, u_n$ and all predicates $P \in \Phi^{\mathcal{N}}$.