Assignment 2

1. Recall the definition of a reduction order.

   (a) An order on terms (atoms, clauses) is a **rewrite order** if it is a monotonic and closed under substitutions.

   (b) An order on terms (atoms, clauses) is a **reduction order** if it is a well-founded rewrite order.

   Prove that a reduction order total on ground literals satisfies the property: for each literal A, $\neg A \succ A$.

2. Recall the definition of a lexicographic path order.

   Let Σ be a finite signature and $>$ a strict order on Σ. The **lexicographic path order** $>_{lpo}$ on terms (or literals) induced by $>$ is defined as follows: $s >_{lpo} t$ iff

   (a) $t \in Var(s)$ and $s \neq t$,

   (b) $s = f(s_1, \ldots, s_m)$, $t = g(t_1, \ldots, t_n)$ and

      i. there is i, $1 \leq i \leq m$, such that $s_i \geq_{lpo} t$ or

      ii. $f > g$ and $s >_{lpo} t_j$, for all j, $1 \leq j \leq n$ or

      iii. $f = g$, there is i, $1 \leq i \leq m$, such that $s_1 = t_1, \ldots, s_{i-1} = t_{i-1}$ and $s_i >_{lpo} t_i$ and $s >_{lpo} t_j$ for all j, $i < j \leq n$,

   Assume that a first order signature contains only connectives ($\wedge$, $\neg$), finitely many functions and predicate symbols. How to define a total order on the signature in such a way that
   a lexicographic order on ground clauses induced by this total order on a signature is an admissible order.