Assignment 3

1. Prove the following statement: If a unification problem G is solvable, then it has an idempotent most general unifier.

   HINT:

   (a) Show that for each unifiable G, it can be transformed into a unification problem in a solved form, in such a way, that this transformation preserves the set of unifiers of G.
   G is in a solved form if $G = \{x_1 =?t_1, \ldots, x_n =?t_n\}$, where $x_1, \ldots, x_n$ are pairwise different variables and no x_i appears in $t_1, \ldots, t_n$.

   (b) If G is in a normal form, we define $\vec{G} := \{x_1 \mapsto t_1, \ldots, x_n \mapsto t_n\}$ Show that:

      i. for each unifier of G, $\vec{G}\,\sigma = \sigma$,

      ii. $\vec{G}\vec{G} = \vec{G}$

2. Prove Lemma 2.2 for ordered resolution $\mathcal{O}$: Let N be a set of ground clauses saturated with inferences from $\mathcal{O}$, such that $[] \notin N$. Let I_N be a Herbrand interpretation constructed as in Definition 3.1. Then $I_N \not\models C$ implies that $I_C \not\models C$.