Assignment 4

1. Prove refutation soundness and completeness of ordered hyperresolution with ordered positive factoring. (Hint: try to follow the proof for the system $O$.)

2. A Horn clause is a clause with at most one positive literal. Prove the refutation soundness and completeness of the following rule for a set of Horn clauses:

   **Positive Unit Resolution:**

   \[
   \begin{array}{c}
   A \quad D \lor \lnot B \\
   \hline
   D_{\sigma}
   \end{array}
   \]

   where $\sigma = mgu(A, B)$.

3. Unit Resolution is a binary resolution where one of the premises must be a unit clause (i.e. a clause containing one literal). Using Unit Resolution prove that $\exists x_5, \lnot S(x_5) \land R(x_5)$ follows from the following set of universally quantified clauses:

   \[
   \{ \lnot S(x_1) \lor \lnot P(x_1, b), \lnot P(a, x_2) \lor Q(x_2, x_2), \lnot Q(x_3, y) \lor R(x_3) \lor S(x_3), P(a, x_4) \}.
   \]