Lehrstuhl für Automatentheorie
Institut für Theoretische Informatik, TU Dresden
Prof. Dr. F. Baader

Nöthnitzer Str. 46
01187 Dresden
Tel.: 0351/463–39167

3. Exercises for the Course 'Description Logics'

Exercise 10:

Let $\mathcal{A}$ be the ABox consisting of the following assertions:

$$
\begin{array}{rr}
\mathsf{likes}(\mathsf{Ralf}, \mathsf{Claudia}) & \mathsf{likes}(\mathsf{Ralf}, \mathsf{Peter}) \\
\mathsf{is\text{-}neighbor\text{-}of}(\mathsf{Claudia}, \mathsf{Peter}) & \mathsf{is\text{-}neighbor\text{-}of}(\mathsf{Peter}, \mathsf{Andrea}) \\
\mathsf{Blond}(\mathsf{Claudia}) & \neg\mathsf{Blond}(\mathsf{Andrea})
\end{array}
$$

(a) Does $\mathcal{A}$ have a model?

(b) Is Ralf an instance of the concept $\exists\mathsf{likes}.(\mathsf{Blond} \sqcap \exists\mathsf{is\text{-}neighbor\text{-}of}.\neg\mathsf{Blond})$ in all models of $\mathcal{A}$?

(c) Is Ralf an instance of the concept $\exists\mathsf{likes}.(\exists\mathsf{is\text{-}neighbor\text{-}of}.\ (\forall\mathsf{is\text{-}neighbor\text{-}of}.\ \neg\mathsf{Blond}))$ in all models of $\mathcal{A}$?

Exercise 11:

Let $\mathcal{T}$ be defined as follows:

$$
\begin{array}{rrcl@{\qquad}rcl}
\mathcal{T} = \{ & A & \sqsubseteq & \exists r.\neg B & C & \sqsubseteq & \forall r.(B \sqcup D) \\
& A \sqcap B & \sqsubseteq & D & D & \sqsubseteq & E_1 \sqcap E_2 \\
& E & \equiv & E_1 \sqcap E_2 \sqcap E_3\} & & &
\end{array}
$$

Which of the following subsumption relations holds? Explain.

(a) $A \sqcap C \sqsubseteq \exists r.\neg B \sqcap D$

(b) $E \sqsubseteq E_1 \sqcap E_2$

(c) $\neg D \sqsubseteq \neg E_1 \sqcap \neg E_2$

(d) $\neg E_3 \sqcap \neg E_2 \sqsubseteq E$

(e) $A \sqcap \exists r.B \sqsubseteq \forall r.\top$

Exercise 12:

Prove claim 7. from Theorem 2.17: $\mathcal{K}$ is consistent iff $\mathcal{A} \not\models \bot(a_{new})$, where a_{new} is a new individual w.r.t. ABox $\mathcal{A}$.

Exercise 13:

In the lecture we saw that bisimulations allow to compare the expressive powers of DLs.

(a) Extend the notion of bisimulation relation to $\mathcal{ALCN}$.

(b) Prove that $\mathcal{ALCN}$ is bisimulation invariant for the bisimulation relation defined in (a).

(c) Prove that $\mathcal{ALCQ}$ is more expressive than $\mathcal{ALCN}$.

Exercise 14:

Let $\rho_1 \subseteq \Delta^{\mathcal{I}_1} \times \Delta^{\mathcal{I}_2}$ and $\rho_2 \subseteq \Delta^{\mathcal{I}_2} \times \Delta^{\mathcal{I}_3}$ be bisimulation relations. Prove that bisimulations are closed under

(a) composition (e.g., $\rho_3 = \rho_2 \circ \rho_1$ is a bisimulation)

(b) union (e.g., $\rho_3 = \rho_1 \cup \rho_2$ is a bisimulation).