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4. Exercises for the Course 'Description Logics'

Exercise 14:

Let $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ be a consistent knowledge base. We write $C \sqsubseteq_{\mathcal{K}} D$ if $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ for all models $\mathcal{I}$ of $\mathcal{K}$. Prove that for all $\mathcal{ALC}$-concepts C and D, we have $C \sqsubseteq_{\mathcal{K}} D$ iff $C \sqsubseteq_{\mathcal{T}} D$.

Hint: Use disjoint unions.

Exercise 15:

Show the following claim:
If a concept C is satisfiable w.r.t. an $\mathcal{ALC}$-TBox $\mathcal{T}$, then for all $n \geq 1$ there is a model $\mathcal{I}_n$ of $\mathcal{T}$ such that: $|C^{\mathcal{I}_n}| \geq n$.

Exercise 16:

In the lecture we saw that $\mathcal{ALC}$ has the finite model property and the tree model property. Show that these properties hold in combination in case the TBox is empty. In particular show the following:

(a) For a concept D let $rd(D)$ denote the role-depth, i.e. the maximal number of nested quantifiers. For an $\mathcal{ALC}$-concept C the role-depth $rd(C)$ is bounded by the concept size $|C|$.

(b) For a tree model $\mathcal{I}$, let $\mathcal{I}_{|n}$ denote the model cut at depth n. Let $\mathcal{I}$ be a tree model with root d and d' the root of $\mathcal{I}_{|n}$. Then for every $\mathcal{ALC}$-concept C with $rd(C) \leq n$ we have, $d \in C^{\mathcal{I}}$ iff $d' \in C^{\mathcal{I}_{|n}}$.

(c) Every $\mathcal{ALC}$-concept C has a tree model of depth $\leq |C|$.

Exercise 17:

Prove or refute the following claim:
Given an $\mathcal{ALC}$-concept C and an $\mathcal{ALC}$-TBox $\mathcal{T}$. If $\mathcal{I}$ is an interpretation and $\mathcal{J}$ its filtration w.r.t. Sub(C) $\cup$ Sub$(\mathcal{T})$, then the relation $\rho = \{(d, [d]) \mid d \in \Delta^{\mathcal{I}}, [d] \in \Delta^{\mathcal{J}},\ d \simeq [d]\}$ is a bisimulation.